Worksheet 20

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1. Solve the differential equation

$$t\ln(t)\frac{dr}{dt} + r = te^t.$$

2. Solve the initial-value problem

$$(x^2+1)\frac{dy}{dx} + 3x(y-1) = 0,\ y(0) = 2.$$

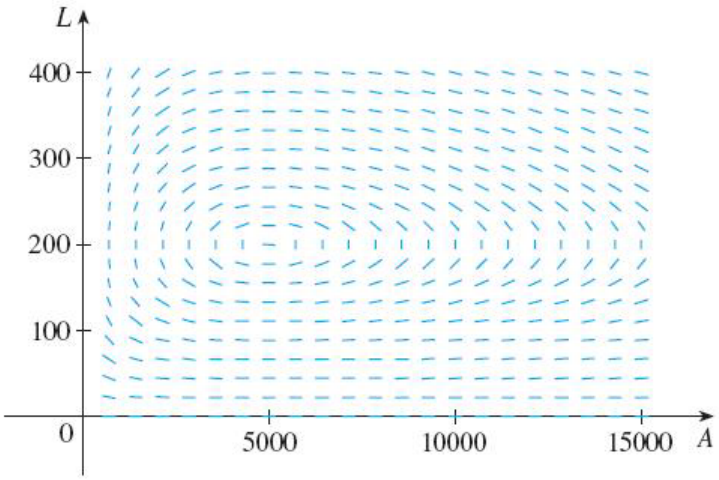
3. Populations of aphids and ladybugs are modeled by the equations

$$\frac{dA}{dt} = 2A - 0.01AL$$
$$\frac{dL}{dt} = -0.5L + 0.0001AL$$

   (a) Explain the picture on the next page.

   (b) Find the equilibrium solutions and explain their significance.

   (c) Find an expression for dL/dA.



(d) Use the direction field below to sketch a phase portrait. What do the phase trajectories have in common?

(e) Suppose at time $t = 0$ there are 1000 aphids and 200 ladybugs. Draw the corresponding phase trajectory and use it to describe how both populations change.

(f) Use part (e) to make rough sketches of the aphid and ladybug populations as functions of t. How are the graphs related to each other?

4. A tank contains 100L of water. A solution with a salt concentration of 0.4kg/L is added at a rate of 5L/min. The solution is kept mixed and is drained from the tank at a rate of 3L/min. If $y(t)$ is the amount of salt (in kilograms) after t minutes, show that y satisfies the differential equation

$$\frac{dy}{dt} = 2 - \frac{3y}{100 + 2t}.$$

Solve this equation and find the concentration after 20 minutes.

5. A Bernoulli differential equation is of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n.$$

For $n \neq 1$, show that the substitution $u = y^{1-n}$ transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x).$$

Use this method to solve

$$xy' + y = -xy^2.$$