Math 1B, Section 110, Fall '09
Quiz 1, September 2

Name: ________________________________
SID: ________________________________

Show all work clearly and in order! You have 15 minutes to take this quiz.

1. (3 points) First make a substitution and then use integration by parts to evaluate the integral
$$\int t^3 e^{-t^2} dt.$$

Solution. The substitution $y = t^2$ yields $dy = 2t \cdot dt$ and
$$\int t^3 e^{-t^2} dt = \frac{1}{2}\int t^2 e^{-t^2}(2t \cdot dt) = \frac{1}{2}\int y e^{-y} dy.$$
Letting $u = y$, $dv = e^{-y}dy$, $v = -e^{-y}$ we get
$$\int y e^{-y} dy = -ye^{-y} + \int e^{-y} dy = -ye^{-y} - e^{-y} + C.$$
It follows that
$$\int t^3 e^{-t^2} dt = \frac{1}{2}(-t^2 e^{-t^2} - e^{-t^2}) + C.$$
□

2. (a) (2 points) Prove the reduction formula
$$\int \sin^n(x)dx = -\frac{1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx.$$
(b) (2 points) Evaluate $\int \sin^3(x)dx$.

Solution. See Stewart, Example 6, Section 7.1, and the solutions to Section 108's quiz. □

3. (3 points) Evaluate
$$\int_0^1 \sqrt{1-x^2}dx.$$
(Hint: Let $u = \sqrt{1-x^2}$, $dv = dx$, and use integration by parts. Alternatively, make the substitution $x = \cos(y)$ and use the reduction formula 2(a).)

Solution. See the solutions to Section 108's quiz. □