Math 1B, Section 110, Fall '09
Quiz 5, October 7

Name: __________________________________
SID: __________________________________

Show all work clearly and in order! You have 15 minutes to take this quiz.

1. (4 points) Determine whether the series

$$\sum_{n=1}^{\infty}\left(\frac{2}{3^n}+\frac{3}{n^2+3n}\right)$$

is convergent or divergent. If it is convergent, find its sum.

Solution. We have

$$\sum_{n=1}^{\infty}\frac{2}{3^n}=\frac{2}{3}\cdot\frac{1}{1-\frac{1}{3}}=1,$$

and

$$\sum_{n=1}^{\infty}\frac{3}{n^2+3n}=\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+3}\right)\overset{\text{telescoping}}{=}\frac{1}{1}+\frac{1}{2}+\frac{1}{3}=\frac{11}{6}.$$

Therefore

$$\sum_{n=1}^{\infty}\left(\frac{2}{3^n}+\frac{3}{n^2+3n}\right)=1+\frac{11}{6}=\frac{17}{6}.$$

□

2. (3 points) Determine the values of p for which the series

$$\sum_{n=1}^{\infty}\frac{1}{n^p}$$

is convergent.

Solution. Answer: $p>1$. See Stewart, Example 2, Section 11.3. □

3. (3 points) Show that if $a_n>0$ and $\lim_{n\to\infty} n\sin(a_n)\neq 0$, then $\sum_{n=1}^{\infty} a_n$ is divergent.

(*Hint*: First show that $\sum\sin(a_n)$ is divergent by comparing it to the harmonic series, and then conclude that $\sum a_n$ is also divergent.)

Solution. We have

$$0\neq\lim_{n\to\infty} n\sin(a_n)=\lim_{n\to\infty}\frac{\sin(a_n)}{\frac{1}{n}}.$$

Using the limit comparison test and the fact that $\sum \frac{1}{n}$ is divergent, it follows that $\sum \sin(a_n)$ is also divergent.

If $\sum a_n$ was convergent, it would follow that $\lim_{n\to\infty} a_n = 0$. Therefore

$$\lim_{n\to\infty} \frac{\sin(a_n)}{a_n} = \lim_{x\to 0} \frac{\sin(x)}{x} \overset{\text{l'Hôpital}}{=} \lim_{x\to 0} \frac{\cos(x)}{1} = 1,$$

and using the comparison test once again we would get that $\sum \sin(a_n)$ is convergent, contradicting the conclusion of the preceding paragraph. In conclusion, $\sum a_n$ is divergent. $\square$