7.1

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int \frac{1}{x}\,dx = \ln|x| + C$

$\int e^x\,dx = e^x + C$

$\int a^x\,dx = \frac{a^x}{\ln a} + C$

$\int \sin x = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \csc x \cot x\,dx = -\csc x + C$

$\int \sinh x\,dx = \cosh x + C$

$\int \cosh x\,dx = \sinh x + C$

$\int \tan x\,dx = \ln|\sec x| + C$

$\int \cot x\,dx = \ln|\sin x| + C$

$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C, a$

Logarithmic
Inverse trig.
Algebraic
Trigonometric
Exponential

$\int u\,dv = u \cdot v - \int v\,du$

u = the first one in this list!

7.3

$\sqrt{a^2-x^2}$, sub. $x = |a|\sin\theta$, $dx = |a|\cos\theta\,d\theta$ $\quad \sqrt{a^2-x^2} = |a|\cos\theta$

$\sqrt{a^2+x^2}$, sub $x = |a|\tan\theta$, $dx = |a|\sec^2\theta\,d\theta$ $\quad \sqrt{a^2+x^2} = |a|\sec\theta$

$\sqrt{x^2-a^2}$, sub $x = |a|\sec\theta$, $dx = |a|\tan\theta\sec\theta\,d\theta$ $\quad \sqrt{x^2-a^2} = |a|\tan\theta$

7.2

$\int \sin^m x \cos^n x\,dx$

(a) $n = 2k+1$, $\cos^2 x = 1 - \sin^2 x$

$\int \sin^m x \cos^{2k+1} x\,dx = \int \sin^m x (\cos^2 x)^k \cdot \cos x\,dx$

$= \int \sin^m x (1-\sin^2 x)^k \cos x\,dx = \int u^m (1-u^2)^k\,du$

Use sub. $u = \sin x \Rightarrow du = \cos x\,dx$

(b) $m = 2k+1$, $\sin^2 x = 1 - \cos^2 x$

$\int \sin^{2k+1} x \cos^n x\,dx = \int (\sin^2 x)^k \cos^n x \sin x\,dx$

$= \int (1-\cos^2 x)^k \cos^n x \cdot \sin x\,dx = -\int (1-u^2)^k \cdot u^n\,du$

Use sub. $u = \cos x \Rightarrow du = -\sin x\,dx$

(c) $m = 2k$, $n = 2\ell$, $\sin^2 x = \frac{1-\cos 2x}{2}$, $\cos^2 x = \frac{1+\cos 2x}{2}$

$\int (\sin^2 x)^k (\cos^2 x)^\ell\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^k \cdot \left(\frac{1+\cos 2x}{2}\right)^\ell dx$

$\int \tan^m x \sec^n x\,dx$

(a) $n = 2k$, $\sec^2 x = 1 + \tan^2 x$

$\int \tan^m x \sec^{2k} x\,dx = \int \tan^m x (\sec^2 x)^{k-1} \cdot \sec^2 x\,dx$

$= \int \tan^m x (1+\tan^2 x)^{k-1} \sec^2 x\,dx$

Sub. $u = \tan x$, $du = \sec^2 x\,dx$

$= \int u^m (1+u^2)^{k-1}\,du$

(b) $m = 2k+1$, $\tan^2 x = \sec^2 x - 1$

$\int \tan^{2k+1} x \sec^n x\,dx =$

$= \int (\tan^2 x)^k \sec^{n-1} x \tan x \sec x\,dx =$

$= \int (\sec^2 x - 1)^k \sec^{n-1} x \tan x \sec x\,dx =$

Sub. $u = \sec x$, $du = \tan x \sec x\,dx$

$= \int (u^2-1)^k u^{n-1}\,du$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$

$\sin(a+b) = \sin a \cos b + \sin b \cos a$

$\sin(a-b) = \sin a \cos b - \sin b \cos a$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\sin 2a = 2 \sin a \cos b$

$\cos 2a = 2\cos^2 a - 1 = 1 - 2\sin^2 a$

$\int \tan x\,dx = \ln|\sec x| + C$

$\sin a \cos b = \frac{1}{2}[\sin(a-b) + \sin(a+b)]$ | $\sin a \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$ | $\cos a \cos b = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$

7.4

$\int f(x)\,dx$, $f(x) = \frac{P(x)}{Q(x)}$, P, Q polynomials. Long division: $\frac{P(x)}{Q(x)} = S(x) + \frac{R(x)}{Q(x)}$, $\deg(R) < \deg(Q)$

I. $Q(x) = (a_1x+b_1)\dots(a_kx+b_k)$. Find $A_1 \dots A_k$ s.t. $\frac{R(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \dots + \frac{A_k}{a_kx+b_k}$

E.g. $\frac{x}{(x+1)(x-1)} = \frac{a}{x+1} + \frac{b}{x-1}$ $\quad (a = \frac{1}{2}, b = \frac{1}{2})$

II. $Q(x) = (a_1x+b_1)^{r_1}\dots(a_kx+b_k)^{r_k}$. Find A_{ij} s.t. $\frac{R(x)}{Q(x)} = \frac{A_{11}}{a_1x+b_1} + \frac{A_{12}}{(a_1x+b_1)^2} + \dots + \frac{A_{1r_1}}{(a_1x+b_1)^{r_1}} + \frac{A_{21}}{(a_2x+b_2)} + \frac{A_{22}}{(a_2x+b_2)^2} + \dots + \frac{A_{2r_2}}{(a_2x+b_2)^{r_2}} + \dots$

E.g. $\frac{x^2-x-1}{x^2(x+1)^2} = \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x+1} + \frac{d}{(x+1)^2}$

$a = 1, b = -1, c = -1, d = 1$

III. Q has irr. factor ax^2+bx+c, $b^2-4ac<0$ $\quad \frac{R(x)}{Q(x)} = \dots + \frac{Ax+B}{ax^2+bx+c} \rightsquigarrow$ complete the square + sub $\boxed{y = x + \frac{b}{2a}}$

$ax^2+bx+c = a\left[\left(x+\frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a^2}\right]$

IV. Q has factor $(ax^2+bx+c)^r$, b^2-4ac $\quad \frac{R(x)}{Q(x)} = \dots + \frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \dots + \frac{A_rx+B_r}{(ax^2+bx+c)^r}$

Rationalizing sub $u=\sqrt[n]{g(x)}$ $u^n = g(x)$ $nu^{n-1}du = g'(x)dx$ E.g. $\int \frac{\sqrt{x+4}}{x}dx$ $u=\sqrt{x+4}$ $u^2=x+4$ $2udu$

Weierstrass sub

$\int f(x)dx$, $f(x)=\frac{P(\sin x,\cos x)}{Q(\sin x,\cos x)}$, P, Q polynomial expressions. Sub. $t=\tan\frac{x}{2}$, $\frac{2dt}{1+t^2}=dx$, $\sin x=\frac{2t}{1+t^2}$, $\cos x=\frac{1-t^2}{1+t^2}$

E.g. $\int\frac{dx}{3\sin x-4\cos x} = \int \frac{1}{\frac{6t}{1+t^2}-\frac{4(1-t^2)}{1+t^2}}\cdot\frac{2dt}{1+t^2}\ldots$

$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$ | $\int\frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left|x+\sqrt{x^2\pm a^2}\right|$

Simpson: n has to be even!!!

$\int_a^b f(x)dx \simeq S_n = \frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+\ldots+4f(x_{n-3})+2f(x_{n-2})+4f(x_{n-1})+f(x_n)]$

$K\geq|f^{(4)}(x)|$ for $a\le x\le b$

$E_S = \left|\int_a^b f(x)dx - S_n\right| \le \frac{K(b-a)^5}{180n^4}$

7.7 $a=x_0\ x_1\ x_2 \ldots x_{n-1}\ x_n=b$ n intervals, $\Delta x=\frac{b-a}{n}$

Midpoint $\int_a^b f(x)dx \simeq M_n = \Delta x\left(f(\bar{x}_1)+\ldots+f(\bar{x}_n)\right)$, $\bar{x}_i = \frac{x_{i-1}+x_i}{2}$

$K\geq|f''(x)|$ $a\le x\le b$ $E_M = \left|\int_a^b f(x)dx - M_n\right| \le \frac{K(b-a)^3}{24n^2}$

Trapezoid $\int_a^b f(x)dx \simeq T_n = \frac{\Delta x}{2}\left(f(x_0)+2f(x_1)+\ldots+2f(x_{n-1})+f(x_n)\right)$

$K\geq|f''(x)|$ $a\le x\le b$ $E_T = \left|\int_a^b f(x)dx - T_n\right| \le \frac{K(b-a)^3}{12n^2}$

Note: Simpson's Rule gives the precise value for $\int_a^b f(x)dx$ when f is polynomial of degree … (because $f^{(4)}(x)=0$)

7.8 a real: $\lim_{t\to 0^+} t^a = \begin{cases}0, & a>0\\ 1, & a=0\\ \infty, & a<0\end{cases}$ | a real: $\lim_{t\to0^+} t^a\ln t = \begin{cases}0, & a>0\\ -\infty, & a\le 0\end{cases}$ | $\int_1^\infty \frac{dx}{x^p}$ is $\begin{cases}\text{conv}, & p>1\\ \text{div}, & p\le 1\end{cases}$ | $\int_0^1\frac{dx}{x^p}$ is $\begin{cases}\text{div}, & p\geq 1\\ \text{conv}, & p<1\end{cases}$

8 $ds=\sqrt{(dx)^2+(dy)^2} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}dx = \sqrt{\left(\frac{dx}{dy}\right)^2+1}\,dy$

8.1 $y=f(x)$ $a\le x\le b$ Arc length $L=\int_a^b\sqrt{1+(f'(x))^2}dx$

$x=g(y)$ $c\le y\le d$ $L=\int_c^d\sqrt{1+(g'(y))^2}dy$

8.2 Rotate about x-axis: $S=2\pi\int y\,ds$

$y=f(x)$ $a\le x\le b$ $S=2\pi\int_a^b f(x)\sqrt{1+[f'(x)]^2}dx$

$x=g(y)$ $c\le y\le d$ $S=2\pi\int_c^d y\sqrt{1+[g'(y)]^2}dy$

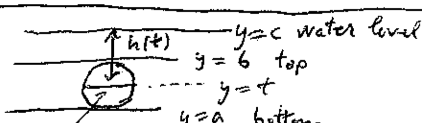
Rotate about y-axis $S=2\pi\int x\,ds$

$y=f(x)$ $a\le x\le b$ $S=2\pi\int_a^b x\sqrt{1+[f'(x)]^2}dx$

$x=g(y)$ $c\le y\le d$ $S=2\pi\int_c^d g(y)\sqrt{1+[g'(y)]^2}dy$

8.3 Hydrostatic force

Completely submerged:

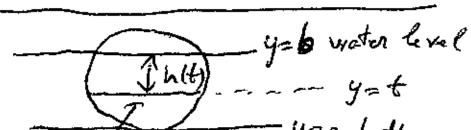


$F=\rho g\int_a^b \ell(t)(c-t)\,dt$

$\ell(t)$ = length of cross section, $h(t)=c-t$ = depth

You get to choose the coordinate axes, so can take $a=0$ or $b=0$ or $c=0$.

Partially submerged:



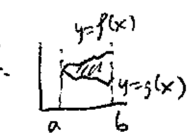
$F=\rho g\int_a^b\ell(t)(b-t)dt$

$\ell(t)$ = length of cross section, $h(t)=b-t$ = depth

You get to choose the coordinate axes. Can take $a=0$ or $b=0$

8.3 Center of mass: A = area, M_x = moment wrt x-axis, M_y = — " — y-axis. $\bar{x}=\frac{M_y}{A}$ $\bar{y}=\frac{M_x}{A}$

I.



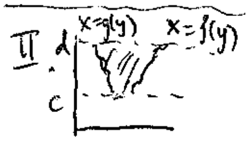
$M_y=\int_a^b x[f(x)-g(x)]dx$

$M_x=\int_a^b\frac{1}{2}[f(x)^2-g(x)^2]dx$

$A=\int_a^b[f(x)-g(x)]dx$ $\bar{x}=\frac{M_y}{A}=\frac{\int_a^b x[f(x)-g(x)]dx}{\int_a^b[f(x)-g(x)]dx}$

$\bar{y}=\frac{M_x}{A}=\frac{\int_a^b\frac{1}{2}[f(x)^2-g(x)^2]dx}{\int_a^b[f(x)-g(x)]}$

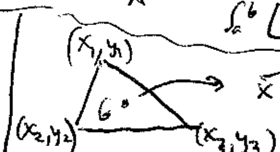
II.



$M_x=\int_c^d y(f(y)-g(y))dy$

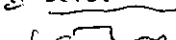
$M_y=\int_c^d\frac{1}{2}[f(y)^2-g(y)^2]dy$

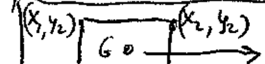
$A=\int_c^d[f(y)-g(y)]dy$



$\bar{x}=\frac{x_1+x_2+x_3}{3}$, $\bar{y}=\frac{y_1+y_2+y_3}{3}$

III Several regions: Each has its M_x^i, M_y^i … $M_x=\sum M_x^i = M_x^1+M_x^2+M_x^3+M_x^4$ $\bar{x}=\frac{M_y}{A}$





$\bar{x}=\frac{x_1+x_2}{2}$

7.1

$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \ n \neq -1$

$\int \frac{1}{x} dx = \ln|x| + C$

$\int e^x dx = e^x + C$

$\int a^x dx = \frac{a^x}{\ln a} + C$

$\int \sin x = -\cos x + C$

$\int \cos x \, dx = \sin x + C$

$\int \sec^2 x \, dx = \tan x + C$

$\int \csc^2 x \, dx = -\cot x$

$\int \sec x \tan x \, dx = \sec x + C$

$\int \csc x \cot x \, dx = -\csc x + C$

$\int \sinh x \, dx = \cosh x + C$

$\int \cosh x \, dx = \sinh x + C$

$\int \tan x \, dx = \ln|\sec x| + C$

$\int \cot x \, dx = \ln|\sin x| + C$

$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

Logarithmic
Inverse trig.
Algebraic
Trigonometric
Exponential

$\int u \, dv = u \cdot v - \int v \, du$

$u =$ the first one in this list!

7.3

$\sqrt{a^2-x^2}$, sub. $x = |a|\sin\theta$, $dx = |a|\cos\theta \, d\theta$; $\sqrt{a^2-x^2} = |a|\cos\theta$

$\sqrt{a^2+x^2}$, sub $x = |a|\tan\theta$, $dx = |a|\sec^2\theta \, d\theta$; $\sqrt{a^2+x^2} = |a|\sec\theta$

$\sqrt{x^2-a^2}$, sub $x = |a|\sec\theta$, $dx = |a|\tan\theta\sec\theta \, d\theta$; $\sqrt{x^2-a^2} = |a|\tan\theta$

7.2 $\int \sin^m x \cos^n x \, dx$

(a) $n = 2k+1$, $\cos^2 x = 1 - \sin^2 x$

$\int \sin^m x \cos^{2k+1} x \, dx = \int \sin^m x (\cos^2 x)^k \cdot \cos x \, dx$

$= \int \sin^m x (1-\sin^2 x)^k \cos x \, dx = \int u^m (1-u^2)^k du$

Use sub. $u = \sin x \Rightarrow du = \cos x \, dx$

(b) $m = 2k+1$, $\sin^2 x = 1 - \cos^2 x$

$\int \sin^{2k+1} x \cos^n x \, dx = \int (\sin^2 x)^k \cos^n x \sin x \, dx$

$= \int (1-\cos^2 x)^k \cos^m x \cdot \sin x \, dx = -\int (1-u^2)^k \cdot u^m du$

Use sub. $u = \cos x \Rightarrow du = -\sin x \, dx$

(c) $m = 2k$, $n = 2\ell$, $\sin^2 x = \frac{1-\cos 2x}{2}$, $\cos^2 x = \frac{1+\cos 2x}{2}$

$\int (\sin^2 x)^k (\cos^2 x)^\ell dx = \int \left(\frac{1-\cos 2x}{2}\right)^k \cdot \left(\frac{1+\cos 2x}{2}\right)^\ell dx$

$\int \tan^m x \sec^n x \, dx$

(a) $n = 2k$, $\sec^2 x = 1 + \tan^2 x$

$\int \tan^m x \sec^{2k} x \, dx = \int \tan^m x (\sec^2 x)^{k-1} \cdot \sec^2 x \, dx$

$= \int \tan^m x (1+\tan^2 x)^{k-1} \sec^2 x \, dx$

sub. $u = \tan x$, $du = \sec^2 x \, dx$

$= \int u^m (1+u^2)^{k-1} du$

(b) $m = 2k+1$, $\tan^2 x = \sec^2 x - 1$

$\int \tan^{2k+1} x \sec^n x \, dx =$

$= \int (\tan^2 x)^k \sec^{n-1} x \tan x \sec x \, dx =$

$= \int (\sec^2 x - 1)^k \sec^{n-1} x \tan x \sec x \, dx =$

Sub. $u = \sec x$, $du = \tan x \sec x \, dx$

$= \int (u^2-1)^k u^{n-1} du$

$\sin(a-b) = \sin a \cos b - \sin b \cos a$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\sin 2a = 2 \sin a \cos b$

$\cos 2a = 2\cos^2 a - 1 = 1 - 2\sin^2 a$

$\int \tan x \, dx = \ln|\sec x|$

$\int \sec x \, dx = \ln|\sec x + \tan x| + C$

$\sin a \cos b = \frac{1}{2}[\sin(a-b) + \sin(a+b)]$ | $\sin a \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$ | $\cos a \cos b = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$

7.4 $\int f(x) dx$, $f(x) = \frac{P(x)}{Q(x)}$, P, Q polynomials. Long division: $\frac{P(x)}{Q(x)} = S(x) + \frac{R(x)}{Q(x)}$, $\deg(R) < \deg(Q)$

I. $Q(x) = (a_1x+b_1)\dots(a_kx+b_k)$. Find $A_1 \dots A_k$ s.t. $\frac{R(x)}{Q(x)} = \frac{A_1}{a_1x+b_1} + \dots + \frac{A_k}{a_kx+b_k}$

E.g. $\frac{x}{(x+1)(x-1)} = \frac{a}{x+1} + \frac{b}{x-1}$ $(a =$; $b =$

II. $Q(x) = (a_1x+b_1)^{r_1} \dots (a_kx+b_k)^{r_k}$. Find A_{ij}'s s.t. $\frac{R(x)}{Q(x)} = \frac{A_{11}}{a_1x+b_1} + \frac{A_{12}}{(a_1x+b_1)^2} + \dots + \frac{A_{1r_1}}{(a_1x+b_1)^{r_1}} + \frac{A_{21}}{(a_2x+b_2)} + \frac{A_{22}}{(a_2x+b_2)^2} + \dots + \frac{A_{2r_2}}{(a_2x+b_2)^{r_2}} + \dots$

E.g. $\frac{x^2-x-1}{x^2(x+1)^2} = \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x+1} + \frac{d}{(x+1)^2}$

$a=1, b=-1, c=-1, d=1$

III. Q has irr. factor ax^2+bx+c, $b^2-4ac<0$: $\frac{R(x)}{Q(x)} = \dots + \frac{Ax+B}{ax^2+bx+c}$ $\rightsquigarrow$ complete the square + sub $\boxed{y = x + \frac{b}{2a}}$

$ax^2+bx+c = a\left[\left(x+\frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a^2}\right]$

IV. Q has factor $(ax^2+bx+c)^r$, b^2-4ac: $\frac{R(x)}{Q(x)} = \dots + \frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \dots + \frac{A_rx+B_r}{(ax^2+bx+c)^r}$

Rationalizing sub $u=\sqrt[n]{g(x)}$ $u^n=g(x)$ $nu^{n-1}du=g'(x)dx$ E.g. $\int\frac{\sqrt{x+4}}{x}dx$ $u=\sqrt{x+4}$ $u^2=x+4$ $2udu=$

Weierstrass sub

$\int f(x)dx$, $f(x)=\frac{P(\sin x,\cos x)}{Q(\sin x,\cos x)}$, P, Q polynomial expressions. Sub. $t=\tan\frac{x}{2}$, $\frac{2dt}{1+t^2}=dx$, $\sin x=\frac{2t}{1+t^2}$, $\cos x=\frac{1-t^2}{1+t^2}$

E.g. $\int\frac{dx}{3\sin x-4\cos x}=\int\frac{1}{\frac{6t}{1+t^2}-\frac{4(1-t^2)}{1+t^2}}\cdot\frac{2dt}{1+t^2}\ldots$

$\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$ | $\int\frac{dx}{\sqrt{x^2\pm a^2}}=\ln\left|x+\sqrt{x^2\pm a^2}\right|$

7.7 | $a=x_0\ x_1\ x_2\ \cdots\ x_{n-1}\ x_n=b$ n intervals, $\Delta x=\frac{b-a}{n}$

Midpoint $\int_a^b f(x)dx\simeq M_n=\Delta x\left(f(\bar{x}_1)+\ldots+f(\bar{x}_n)\right)$, $\bar{x}_i=\frac{x_{i-1}+x_i}{2}$

$K\geq|f''(x)|$ $a\leq x\leq b$ $E_M=\left|\int_a^b f(x)dx-M_n\right|\leq\frac{K(b-a)^3}{24n^2}$

Trapezoid $\int_a^b f(x)dx\simeq T_n=\frac{\Delta x}{2}\left(f(x_0)+2f(x_1)+\ldots+2f(x_{n-1})+f(x_n)\right)$

$K\geq|f''(x)|$ $a\leq x\leq b$ $E_T=\left|\int_a^b f(x)dx-T_n\right|\leq\frac{K(b-a)^3}{12n^2}$

Simpson n has to be even!!!

$\int_a^b f(x)dx\simeq S_n=\frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+\ldots\ldots 4f(x_{n-3})+2f(x_{n-2})+4f(x_{n-1})+f(x_n)\right]$ ...4, 2, 4,

$K\geq|f^{(4)}(x)|$ for $a\leq x\leq b$

$E_S=\left|\int_a^b f(x)dx-S_n\right|\leq\frac{K(b-a)^5}{180n^4}$

Note | Simpson's Rule gives the precise value for $\int_a^b f(x)dx$ when f is polynomial of deg ≤ 3 (because $f^{(4)}(x)=0$)

7.8 | a real $\lim_{t\to0^+}t^a=\begin{cases}0, & a>0\\1, & a=0\\\infty, & a<0\end{cases}$ | a real $\lim_{t\to0^+}t^a\ln t=\begin{cases}0, & a>0\\-\infty, & a\leq0\end{cases}$ | $\int_1^\infty\frac{dx}{x^p}$ is $\begin{cases}\text{conv}, & p>1\\\text{div.}, & p\leq1\end{cases}$ | $\int_0^1\frac{dx}{x^p}$ is $\begin{cases}\text{div}, & p\geq1\\\text{conv}, & p<1\end{cases}$

.1 | $ds=\sqrt{(dx)^2+(dy)^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx=\sqrt{\left(\frac{dx}{dy}\right)^2+1}\,dy$

.1 | $y=f(x)$ $a\leq x\leq b$ Arc length $L=\int_a^b\sqrt{1+(f'(x))^2}dx$

$x=g(y)$ $c\leq y\leq d$ $L=\int_c^d\sqrt{1+(g'(y))^2}dy$

.2 | Rotate about x-axis: $S=2\pi\int y\,ds$

$y=f(x)$ $a\leq x\leq b$ $S=2\pi\int_a^b f(x)\sqrt{1+[f'(x)]^2}dx$

$x=g(y)$ $c\leq y\leq d$ $S=2\pi\int_c^d y\sqrt{1+[g'(y)]^2}dy$

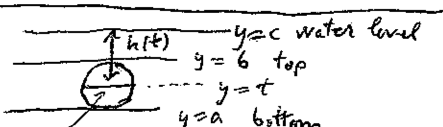
Rotate about y-axis $S=2\pi\int x\,ds$

$y=f(x)$ $a\leq x\leq b$ $S=2\pi\int_a^b x\sqrt{1+[f'(x)]^2}dx$

$x=g(y)$ $c\leq y\leq d$ $S=2\pi\int_c^d g(y)\sqrt{1+[g'(y)]^2}dy$

8.3 | Hydrostatic force

Completely submerged:



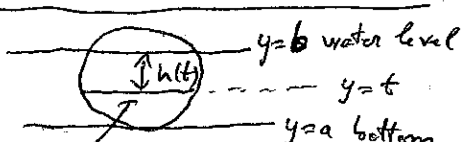
$F=\rho g\int_a^b \ell(t)(c-t)\,dt$

$\ell(t)$ = length of cross section

$h(t)=c-t$ = depth

You get to choose the coordinate axes, so can take $a=0$ or $b=0$ or $c=0$.

Partially submerged:



$F=\rho g\int_a^b \ell(t)(b-t)dt$

$\ell(t)$ = length of cross section

$h(t)=b-t$ = depth

You get to choose the coordinate axes. Can take $a=0$ or $b=0$

8.3 | Center of mass | A = area, M_x = moment wrt x-axis, M_y = — " — y-axis $\bar{x}=\frac{M_y}{A}$ $\bar{y}=\frac{M_x}{A}$

I. $y=f(x)$, $y=g(x)$, a, b

$M_y=\int_a^b x[f(x)-g(x)]dx$

$M_x=\int_a^b\frac{1}{2}[f(x)^2-g(x)^2]dx$

$A=\int_a^b[f(x)-g(x)]dx$ $\bar{x}=\frac{M_y}{A}=\frac{\int_a^b x[f(x)-g(x)]dx}{\int_a^b[f(x)-g(x)]dx}$

$\bar{y}=\frac{M_x}{A}=\frac{\int_a^b\frac{1}{2}[f(x)^2-g(x)^2]dx}{\int_a^b[f(x)-g(x)]}$

II. $x=g(y)$ $x=f(y)$, d, c

$M_x=\int_c^d y(f(y)-g(y))dy$

$M_y=\int_c^d\frac{1}{2}[f(y)^2-g(y)^2]dy$

$A=\int_c^d[f(y)-g(y)]dy$

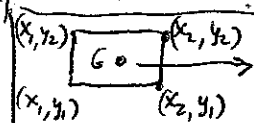
$\bar{x}=\frac{x_1+x_2+x_3}{3}$, $\bar{y}=\frac{y_1+y_2+y_3}{3}$

Several regions Each has its M_x^i, M_y^i, A_i

$M_x=\sum M_x^i=M_x^1+M_x^2+M_x^3+M_x^4$ $\bar{x}=\frac{M_y}{A}$

$M_y=\sum M_y^i=M_y^1+M_y^2+M_y^3+M_y^4$

$A=\sum A_i=A_1+A_2+A_3+A_4$ $\bar{y}=\frac{M_x}{A}$



$\bar{x}=\frac{x_1+x_2}{2}$ $\bar{y}=\frac{y_1+y_2}{2}$