Worksheet 11

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Solve the differential equation or initial-value problem using the method of undetermined coefficients.

1. $y'' + y = e^x + x^3$.
2. $y'' - 4y = e^x \cos(x)$, $y(0) = 1$, $y'(0) = 2$.

Solve the differential equation using the method of variation of parameters.

3. $y'' + y = \sec^3(x)$, $0 < x < \pi/2$.
4. $y'' + 3y' + 2y = \sin(e^x)$.

Solve the differential equation using (a) undetermined coefficients and (b) variation of parameters.

5. $y'' - 2y' - 3y = x + 2$.
6. $y'' - y' = e^x$.

Spring system		Electric circuit	
x	displacement	Q	charge
dx/dt	velocity	$I = dQ/dt$	current
m	mass	L	inductance
c	damping constant	R	resistance
k	spring constant	$1/C$	elastance
$F(t)$	external force	$E(t)$	electromotive force

$$mx'' + cx' + kx = F(t)$$

$$LQ'' + RQ' + \frac{1}{C}Q = E(t)$$

7. A force of 13 N is needed to keep a spring with a 2-kg mass stretched 0.25m beyond its natural length. The damping constant of the spring is $c = 8$. If the mass starts at the equilibrium position with a velocity of 0.5m/s, find its position at time t.

8. Suppose a spring has mass m and spring constant k and let $\omega = \sqrt{k/m}$. Suppose that the damping constant is so small that the damping force is negligible. If an external force $F(t) = F_0 \cos(\omega_0 t)$ is applied, where $\omega_0 \neq \omega$, use the method of undetermined coefficients to show that the motion of the mass is described by the equation
$$x(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t) + \frac{F_0}{m(\omega^2 - \omega_0^2)} \cos(\omega_0 t).$$

9. A series circuit contains a resistor with $R = 24\Omega$, an inductor with $L = 2H$, a capacitor with $C = 0.005F$, and a 12V battery. The initial charge is $Q = 0.001C$ and the initial current is 0. Find the charge and current at time t.

10. Use power series to solve the differential equation

    (a) $y'' + xy' + y = 0$

    (b) $y'' = xy$

    (c) $y'' + x^2y' + xy = 0$, $y(0) = 0$, $y'(0) = 1$