Worksheet 5

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Find a formula for the general term of the sequence

1. $\{2, 7, 12, 17, \cdots\}$
2. $\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \cdots\}$
3. $\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \cdots\}$

Determine whether the sequence converges or diverges. If it converges, find the limit.

4. $a_n = n\cos(n\pi)$
5. $a_n = \dfrac{\sin(2n)}{1+\sqrt{n}}$
6. $a_n = \dfrac{(-3)^n}{n!}$
7. A sequence $\{a_n\}$ is given by $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2 + a_n}$. Show that $\{a_n\}$ is increasing and bounded above by 2. Find $\lim\limits_{n\to\infty} a_n$.
8. A sequence $\{a_n\}$ is given by $a_1 = 2$, $a_{n+1} = \dfrac{1}{3 - a_n}$. Show that $\{a_n\}$ is decreasing and satisfies $0 < a_n \leq 2$. Find $\lim\limits_{n\to\infty} a_n$.

Determine whether the sequence is increasing, decreasing or not monotonic. Is the sequence bounded?

9. $n(-1)^n$
10. $\dfrac{2n-3}{3n+4}$
11. $\dfrac{n}{n^2+1}$

Determine whether the series is convergent or divergent. If it is convergent, find the sum.

12. $\displaystyle\sum_{k=1}^{\infty}(\cos\ 1)^k$
14. $\displaystyle\sum_{n=1}^{\infty}\frac{e^n}{n^4}$
16. $\displaystyle\sum_{n=1}^{\infty}\ln\frac{n}{n+1}$
13. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3}{5^n} - \frac{2}{n}\right)$
15. $\displaystyle\sum_{n=1}^{\infty}\frac{2}{n^2+3n+4}$
17. If the nth partial sum of a series $\displaystyle\sum_{n=1}^{\infty} a_n$ is $s_n = 3 - n2^{-n}$, find a_n and $\displaystyle\sum_{n=1}^{\infty} a_n$.

Find the values of x for which the series converges. Find the sum of the series for those values of x.

18. $\sum_{n=1}^{\infty}(x-4)^n$

19. $\sum_{n=1}^{\infty}\frac{(x+3)^n}{2^n}$

20. $\sum_{n=1}^{\infty}\frac{\cos^n(x)}{2^n}$

21. Find the values of p for which the series

$$\sum_{n=2}^{\infty}\frac{1}{n\ln(n)^p}$$

is convergent.

22. Find all positive values of b for which the series $\sum_{n=1}^{\infty}b^{\ln(n)}$ converges.