Worksheet 6

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Determine whether the series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$.
2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$.
3. $\displaystyle\sum_{n=1}^{\infty} \tan\left(\frac{1}{n^2+3n+4}\right)$.
4. Show that if $a_n > 0$ and $\lim_{n\to\infty} n a_n \neq 0$, then $\sum a_n$ is divergent.
5. Show that if $a_n > 0$ and $\sum a_n$ is convergent, then $\sum \ln(1+a_n)$ is convergent.
6. Show that if $a_n > 0$ and $\sum a_n$ is convergent, then $\sum \sin(a_n)$ is convergent.
7. If $\sum a_n$ and $\sum b_n$ are both convergent series with positive terms, is it true that $\sum a_n b_n$ is also convergent?
8. (Cauchy condensation test) If $\{a_n\}$ is a monotone decreasing sequence, then $\sum a_n$ converges if and only if $\sum 2^n a_{2^n}$ converges.
9. Use the Cauchy condensation test to determine the values of p for which
$$\sum_{n=3}^{\infty} \frac{1}{n\ln(n)(\ln(\ln(n)))^p}$$
is convergent.
10. How many terms of the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln\ n)^2}$ would you need to add to find its sum to within 0.01?

Determine whether the series converges or diverges.

11. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$.
12. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^2+4}}$.
13. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{\pi}{n}\right)$.
14. For which values of p is the series
$$\sum_{n=2}^{\infty} (-1)^{n-1} \frac{(\ln\ n)^p}{n}$$
convergent?

15. How many terms of the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{2^n n}$ would you need to add to find its sum to within 0.001?

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

16. $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$.

17. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$.

18. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+2}}$.

19. $\sum_{n=1}^{\infty} (-1)^n \frac{n!}{n^n}$.

20. $\sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j+5}$.

Test the series for convergence or divergence

21. $\sum_{n=0}^{\infty} \frac{n!}{2 \cdot 5 \cdot 8 \cdots (3n+2)}$.

22. $\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^{n^2}$.

23. $\sum_{n=1}^{\infty} \tan(1/n)$.

24. $\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^n}{n^{4n}}$.

25. $\sum_{n=1}^{\infty} (\sqrt[n]{3} - 1)$.

26. Show that the series $\sum (-1)^{n-1} b_n$, where $b_n = 1/n$ if n is odd and $b_n = 1/n^2$ if n even, is divergent. Why does the Alternating Series Test not apply?

27. Is the series

$$\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)$$

convergent or divergent?

28. For which positive integers k is the following series convergent?

$$\sum_{n=1}^{\infty} \frac{(n!)^k}{(kn)!}.$$