Worksheet 7

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Find the radius of convergence and the interval of convergence of the series

1. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}.$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{10^n x^n}{n^3}.$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}.$

4. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{4^n \ln n}.$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2(2x-3)^n}{2 \cdot 4 \cdot 6 \cdots (2n)}.$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}.$

7. $\displaystyle\sum_{n=1}^{\infty} n!(2x-1)^n.$

8. $\displaystyle\sum_{n=2}^{\infty} \frac{x^{2n}}{n(\ln n)^2}.$

Find a power series representation for the function and determine the interval and radius of convergence.

9. $f(x) = \dfrac{3}{1-x^4}.$

10. $f(x) = \dfrac{1+x}{1-x}.$

11. $f(x) = \dfrac{x+2}{2x^2-x-1}.$

12. $f(x) = \dfrac{x^2}{(1-2x)^2}$

13. $f(x) = \arctan(x/3).$

14. $f(x) = \dfrac{x}{x^2+16}.$

15. $f(x) = \ln(x^2+4).$

16. Starting with the geometric series $\sum_{n=0}^{\infty} x^n$, find the sum of the series

(a) $\displaystyle\sum_{n=1}^{\infty} nx^{n-1} \quad |x| < 1.$

(b) $\displaystyle\sum_{n=1}^{\infty} nx^n \quad |x| < 1.$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}.$

(d) $\displaystyle\sum_{n=2}^{\infty} n(n-1)x^n \quad |x| < 1.$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-n}{2^n}.$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}.$