Worksheet 8

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Determine the Maclaurin series for the function

1. $f(x) = e^x + e^{2x}$.
2. $f(x) = x^2 \tan^{-1}(x^3)$.
3. $f(x) = \dfrac{x}{\sqrt{4+x^2}}$.
4. $f(x) = \sin^2 x$.
5. $\sin^{-1} x$.

Use series to evaluate the limit

6. $\lim\limits_{x\to 0} \dfrac{x - \tan^{-1} x}{x^3}$.
7. $\lim\limits_{x\to 0} \dfrac{1 - \cos\ x}{1 + x - e^x}$.
8. $\lim\limits_{x\to 0} \dfrac{\tan\ x - x}{x^3}$.
9. Determine the Maclaurin series for $f(x) = \sinh\ x$ and prove that it represents $f(x)$ for all values of x.

Find the first three nonzero terms in the Maclaurin series for the function

10. $y = \dfrac{x}{\sin\ x}$.
11. $y = e^x \ln(1 - x)$.

(12-13) (a) Approximate f by a Taylor polynomial with degree n at the number a.

(b) Use Taylor's inequality to estimate the accuracy of the approximation $f(x) \approx T_n(x)$ when x lies in the given interval.

12. $f(x) = \ln(1 + 2x),\ a = 1,\ n = 3,\ 0.5 \le x \le 1.5$.
13. $f(x) = x \sin\ x,\ a = 0,\ n = 4,\ -1 \le x \le 1$.
14. Use the Alternating Series Estimation Theorem or Taylor's Inequality to estimate the range of values of x for which the given approximation is accurate to within the stated error.

$$\cos\ x \approx 1 - \frac{x^2}{2} + \frac{x^4}{24} \quad (|\text{error}| < 0.005)$$