Worksheet 9

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1. Verify that $y = \sin x \cos x - \cos x$ is a solution of the initial-value problem

$$y' + (\tan x)y = \cos^2 x, \quad y(0) = -1$$

    on the interval $-\pi/2 < x < \pi/2$.

2. (a) For what values of k does the function $y = \cos kt$ satisfy the differential equation $4y'' = -25y$?

    (b) For those values of k, verify that every member of the family of functions $y = A \sin kt + B \cos kt$ is also a solution.

3. A function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt} = y^4 - 6y^3 + 5y^2.$$

    (a) What are the constant solutions of the equation?

    (b) For what values of y is y increasing/decreasing?

4. The function with the given graph is a solution of one of the following differential equations. Decide which is the correct equation and justify your answer.
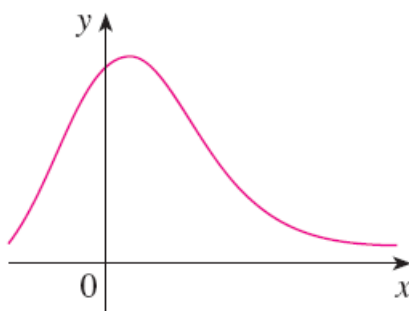
    (a) $y' = 1 + xy$.   (b) $y' = 2xy$.   (c) $y' = 1 - 2xy$.

5. A direction field for the differential equation $y' = x \sin y$ is shown.

    (i) Sketch the graphs of the solutions that satisfy the given initial conditions.

(a) $y(0) = 1$.   (b) $y(0) = 2$.   (c) $y(0) = \pi$.   (d) $y(0) = 4$.   (e) $y(0) = 5$.
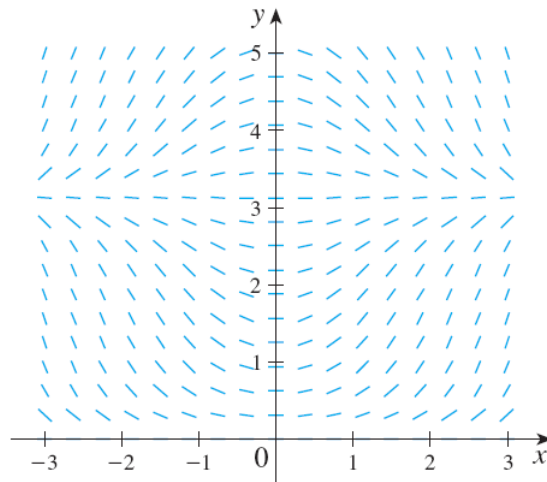
(ii) Find all the equilibrium solutions.

6. Use Euler's method with step size 0.2 to estimate $y(1)$, where $y(x)$ is the solution of the initial-value problem $y' = 1 - xy$, $y(0) = 0$.

Solve the differential equation

7. $\dfrac{dy}{dx} = \dfrac{\sqrt{x}}{e^y}$.

8. $y' = y^2 \sin x$.

Find the solution of the differential equation that satisfies the given condition.

9. $\dfrac{dy}{dx} = \dfrac{y \cos x}{y^2 + 1}$, $y(0) = 1$.

10. $x \cos x = (2y + e^{3y})y'$, $y(0) = 0$.

Find the orthogonal trajectories of the family of curves.

11. $y^2 = kx^3$.

12. $y = \dfrac{x}{1 + kx}$.

13. A vat with 500 gallons of beer contains 4% alcohol (by volume). Beer with 6% alcohol is pumped into the vat at a rate of 5 gal/min and the mixture is pumped out at the same rate. What is the percentage of alcohol after an hour?

14. The air in a room with volume 180 m^3 contains 0.15% carbon dioxide initially. Fresher air with only 0.05% carbon dioxide flows into the room at a rate of 2 m^3/min and the mixed air flows out at the same rate. Find the percentage of carbon dioxide in the room as a function of time. What happens in the long run?