Math 1B, Spring '10
Quiz 1, January 27

1. (3 points) Evaluate
$$\int_1^2 \frac{\ln\ x}{x^2} dx.$$

Proof. Recall the mnemonic we used for integrating by parts: LIATE. Since $\ln(x)$ is a Logarithmic function, and $\frac{1}{x^2}$ is an Algebraic function, we put $u = \ln(x)$, $dv = \frac{1}{x^2}dx$, so that $du = \frac{1}{x}dx$ and $v = \frac{-1}{x}$. From the formula
$$\int_1^2 udv = uv]_1^2 - \int_1^2 vdu$$
we get
$$\begin{aligned}\int_1^2 \frac{\ln\ x}{x^2} dx &= \ln(x)\frac{-1}{x}\bigg]_1^2 - \int_1^2 \frac{-1}{x}\cdot\frac{1}{x}dx \\ &= \frac{-\ln(2)}{2} + \int_1^2 \frac{1}{x^2}dx = \frac{-\ln(2)}{2} - \frac{1}{x}\bigg]_1^2 \\ &= \frac{-\ln(2)}{2} - \frac{1}{2} + \frac{1}{1} = \frac{1-\ln(2)}{2}\end{aligned}$$

□

2. (4 points) First make a substitution and then use integration by parts to evaluate the integral
$$\int \cos(\sqrt{x})dx.$$

Solution. Make the substitution $y = \sqrt{x}$. We have $y^2 = x$ so after differentiation we get $2y \cdot dy = dx$, thus
$$\int \cos(\sqrt{x})dx = 2\int y\cos(y)dy.$$
Use now integration by parts, with $u = y$, $dv = \cos(y)dy$, $v = \sin(y)$:
$$\int y\cos(y)dy = y\sin(y) - \int \sin(y)dy = y\sin(y) + \cos(y) + C.$$
It follows that
$$\int \cos(\sqrt{x})dx = 2(\sqrt{x}\sin(\sqrt{x}) + \cos(\sqrt{x})) + C.$$

□

3. (3 points) Evaluate
$$\int \left(\sin^2(x) + \cos^3(x)\right) dx.$$

Proof. We can split the integral into two parts

$$\int \sin^2(x)dx \text{ and } \int \cos^3(x)dx.$$

To evaluate the first integral, we use the half angle formula $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$.

$$\int \sin^2(x)dx = \frac{1}{2}\int (1-\cos(2x))dx = \frac{1}{2}\left(x - \frac{\sin(2x)}{2}\right) + C.$$

For the second integral, we make the substitution $u = \sin(x)$, so $du = \cos(x)dx$, and use the fact that $\cos^2(x) = 1 - \sin^2(x) = 1 - u^2$. We get

$$\int \cos^3(x)dx = \int \cos^2(x)\cdot\cos(x)dx = \int (1-u^2)du = u - \frac{u^3}{3} + C = \sin(x) - \frac{\sin^3(x)}{3} + C.$$

Putting these calculations together, we obtain

$$\int \left(\sin^2(x) + \cos^3(x)\right) dx = \frac{1}{2}\left(x - \frac{\sin(2x)}{2}\right) + \sin(x) - \frac{\sin^3(x)}{3} + C.$$

$\square$