Math 20550, Practice Exam 2
October 26, 2017

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes..
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 9 pages of the test.
- Each multiple choice question is 6 points, each partial credit problem is 12 points. You will receive 4 extra points.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1. (a)   (b)   (c)   (d)   (e)

2. (a)   (b)   (c)   (d)   (e)

3. (a)   (b)   (c)   (d)   (e)

4. (a)   (b)   (c)   (d)   (e)

5. (a)   (b)   (c)   (d)   (e)

6. (a)   (b)   (c)   (d)   (e)

7. (a)   (b)   (c)   (d)   (e)

8. (a)   (b)   (c)   (d)   (e)

9. (a)   (b)   (c)   (d)   (e)

10. (a)   (b)   (c)   (d)   (e)

Please do NOT write in this box.

Multiple Choice   _____

11.   _____

12.   _____

13.   _____

Extra Points.   _4_____

Total: _____

Multiple Choice

1.(6 pts) Let $f(x, y)$ be a function where $(1, 3)$ and $(-1, 0)$ are critical points. We also know that $f_{xx}(1, 3) = 1, f_{x,y}(1, 3) = 2, f_{yy}(1, 3) = 1$ and $f_{xx}(-1, 0) = 2, f_{x,y}(-1, 0) = -1, f_{yy}(-1, 0) = 3$. Using the second derivative test classify the points $(1, 3)$ and $(-1, 0)$.

(a)   both are local minimums

(b) $(1, 3)$ is a saddle point; $(-1, 0)$ is a local minimum

(c) $(1, 3)$ is a saddle point; $(-1, 0)$ is a local maximum

(d)   both are saddle points

(e) $(1, 3)$ is a local maximum; $(-1, 0)$ is a local minimum

2.(6 pts) Use implicit differentiation to find $\partial z / \partial x$ when $xz + z^2 = y$.

(a) $\dfrac{\partial z}{\partial x} = \dfrac{-z}{x + 2z}$

(b) $\dfrac{\partial z}{\partial x} = \dfrac{y}{x + z}$

(c) $\dfrac{\partial z}{\partial x} = \dfrac{-x}{2z}$

(d) $\dfrac{\partial z}{\partial x} = \dfrac{y - z}{x + 2z}$

(e) $\dfrac{\partial z}{\partial x} = \dfrac{y - x}{2z}$

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3.(6 pts) Find the directional derivative of $f(x,y) = xe^{-2y}$ at the point $(1,0)$ in the direction $\langle 1,3 \rangle$.

(a) $\dfrac{-5}{\sqrt{10}}$ (b) $\ 0$ (c) $\ -4$ (d) $\dfrac{-1}{2}$ (e) $\ \sqrt{10}$

4.(6 pts) Consider the two surfaces $\mathcal{S}_1 : y + z = 4$ and $\mathcal{S}_2 : z = 2x^2 + 3y^2 - 12$. Find the tangent line to the intersection curve of $\mathcal{S}_1$ and $\mathcal{S}_2$ at the point $(1, 2, 2)$.

(a) $\langle x, y, z \rangle = \langle 11t, -4t, 4t \rangle + \langle 1, 2, 2 \rangle$

(b) $\langle x, y, z \rangle = \langle -11t, 4t, -4t \rangle + \langle -1, -2, -2 \rangle$

(c) $\langle x, y, z \rangle = \langle -11t, 4t, -4t \rangle + \langle 1, 2, 2 \rangle$

(d) $\langle x, y, z \rangle = \langle -13t, 4t, -4t \rangle + \langle -1, -2, -2 \rangle$

(e) $\langle x, y, z \rangle = \langle -13t, 4t, -4t \rangle + \langle 1, 2, 2 \rangle$

5.(6 pts) Let $f(x, y)$ be a function of $x(s, t) = st$ and $y(s, t) = 2s + t$. If you know that $f_x(1, 3) = 2$ and $f_y(1, 3) = -3$ then what is $\partial f/\partial s$ at when $s = 1$ and $t = 1$?

(a) -1 (b) not enough information to determine the value

(c) 3 (d) -4

(e) 0

6.(6 pts) Find a point on the surface $z = x^2 - y^3$ where the tangent plane is parallel to the plane $x + 3y + z = 0$.

(a) no such point exists   (b) $(-1/2, 1, -3/4)$ (c) $(1, 1, 0)$

(d) $(-1/2, 1, 1)$ (e) $(-1/2, 1, -5/2)$

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7.(6 pts) Let f be the function $f(x, y, z) = \sin(xyz)$. From the point $(1, 1, 0)$ in which direction should one move in order to attain the maximum rate of change.

(a) $\dfrac{1}{\sqrt{2}}\langle 1, 1, 0\rangle$ (b) $\langle 0, 0, 1\rangle$ (c) $\dfrac{1}{\sqrt{2}}\langle 0, 0, 1\rangle$ (d) $\langle 0, 0, 0\rangle$ (e) $\langle 1, 1, 1\rangle$

8.(6 pts) Find the absolute maximum value of the function $f(x, y, z) = xy + \dfrac{z^2}{2}$ under the two constraints $y - 2z = 0$ and $x + z = -1$.

(a) $\dfrac{22}{9}$ (b) $\dfrac{-2}{9}$ (c) $\dfrac{2}{3}$ (d) $\dfrac{2}{9}$ (e) $\dfrac{-1}{2}$

9.(6 pts) Which of the following integrals represents the volume of the solid delimited by $y = 0$, $y = 1$, $x = 0$, $x = 2$, $z = 0$ and $z = x^2y + y^3$.

(a) $\int_0^2 \int_0^1 (x^2y + y^3)\, dy\, dx$

(b) $\int_0^2 \int_0^1 (-x^2y - y^3)\, dx\, dy$

(c) $\int_0^2 \int_0^1 (-x^2y - y^3)\, dy\, dx$

(d) $\int_0^2 \int_0^1 (x^2y + y^3)\, dx\, dy$

(e) $\int_1^2 \int_0^1 (x^2y + y^3)\, dy\, dx$

10.(6 pts) Compute $\iint_R 24xy\, dA$ where R is the region bounded by $x = 1, x = 2, y = x$, and $y = x^2$.

(a) 62   (b) 128   (c) 64   (d) 48   (e) 81

Partial Credit
You must show your work on the partial credit problems to receive credit!

11.(12 pts.) Find the absolute maximum and absolute minimum values of the function $f(x, y) = x - 3y$ subject to the constraint $x^2 + 2y^2 = 3$.

12.(12 pts.) Consider the iterated integral $\int_0^2 \int_{y^2}^4 y^3 e^{x^3} \, dx \, dy$.

    (a) Sketch the region of integration.
    (b) Rewrite the integral with the order of integration reversed.
    (c) Compute the value of the iterated integral.

13.(12 pts.) Determine the absolute maximum and minimum values of the function $f(x, y) = x^2 y - xy + x$ on the region $0 \le x \le 2$, $-2 \le y \le 0$.