Department of Mathematics
University of Notre Dame
Math 20580 – Fall 2014

Name: ____________________

Instructor: ______________
& Section

Exam II

October 30, 2014.

This exam is in 2 parts on 9 pages and contains 13 problems worth a total of 96 points. An additional 4 points will be awarded for following the instructions. You have 1 hour and 15 minutes to work on it. No calculators, books, notes, or other aids are allowed. Be sure to write your name on this page and to put your initials at the top of every page in case pages become detached. Good luck!

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Place an × through your answer to each problem.

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MC.	________
11.	________
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Multiple Choice

1. (6 pts.) Let $\mathbf{b_1} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{b_2} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $\mathbf{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$. Find $[\mathbf{x}]_{\mathbf{B}}$.

(a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$ (d) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (e) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

2. (6 pts.) Let $\{\lambda_1, \lambda_2\}$ be two eigenvalues of $\begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}$. Then product of the two eigenvalues, $\lambda_1\lambda_2$, is equal to

(a) 4 (b) $4i$ (c) 8 (d) $-4i$ (e) -8

3. (6 pts.) Which of the following is a subspace of $\mathbb{R}^2$?

(a) All the sets above.

(b) Union of two distinct straight lines passing through 0.

(c) Upper half plane.

(d) Orthogonal complement of a set in $\mathbb{R}^2$.

(e) The set of all the eigenvectors of a 2×2 matrix.

4. (6 pts.) Let $A = \begin{bmatrix} -2 & 4 & 1 & 4 \\ -3 & 6 & -1 & 1 \\ 1 & -2 & 2 & 3 \\ 2 & -4 & 5 & 9 \end{bmatrix}$. Find a basis for Row($A$).

(a) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(c) All the rows of A

(d) Orthogonal complement of a set in $\mathbb{R}^2$.

(e) $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

5. (6 pts.) Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, and let $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$ be a basis of $\mathbb{R}^2$. Find the $\mathcal{B}$-matrix for the transformation $\mathbf{x} \to A\mathbf{x}$.

(a) $\begin{bmatrix} -6 & -7 \\ 5 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ (e) $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}$

6. (6 pts.) Let $A = \begin{bmatrix} 1 & 3 & -2 & 5 & 3 \\ 0 & 1 & -1 & 2 & 1 \\ 2 & 1 & 1 & 2 & 3 \end{bmatrix}$. Which of the following is equal to the dimension of the column space of A?

(a) 4 (b) 2 (c) 1 (d) 3 (e) 5

7. (6 pts.) Which of the following statements is true of the length of vectors?

(a) $\mathbf{u} \cdot \mathbf{v} = -\mathbf{v} \cdot \mathbf{u}$

(b) $\mathbf{u} \cdot \mathbf{u} = \| \mathbf{u} \|^2$

(c) $|\mathbf{u} \cdot \mathbf{v}| = \| \mathbf{u} \| \| \mathbf{v} \|$

(d) The length of $\frac{\mathbf{u}}{\|\mathbf{u}\|}$ is $\frac{1}{\|\mathbf{u}\|}$.

(e) $\| \mathbf{u} + \mathbf{v} \|^2 = \| \mathbf{u} \|^2 + \| \mathbf{v} \|^2$

8. (6 pts.) Compute the orthogonal projection of $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ onto the line through $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ and the origin

(a) $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ (c) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ (d) $\frac{9}{5}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ (e) $\frac{1}{5}\begin{bmatrix} 1 \\ 7 \end{bmatrix}$

9. (6 pts.) Let A and B be two $n \times n$ matrices with $\det(A) = 0$. Which of the following statements is not true?

(a) Rank $A < n$

(b) A is not invertible.

(c) 0 is an eigenvalue of A

(d) Columns of A span $\mathbb{R}^n$

(e) $\det(AB) = 0$

10. (6 pts.) Let $\mathcal{B} = \left\{ \begin{bmatrix} 7 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix} \right\}$ be bases for $\mathbb{R}^2$. Find $P_{\mathcal{C} \leftarrow \mathcal{B}}$, the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$.

(a) $\begin{bmatrix} 7 & -3 \\ 5 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ -5 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 22 & -20 \\ 10 & -12 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 1 \\ -5 & 3 \end{bmatrix}$ (e) $\begin{bmatrix} -3 & 1 \\ -5 & 2 \end{bmatrix}$

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (12 pts.) Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$.

(a) Find eigenvalues and the eigenspace corresponding to each eigenvalue.

(b) Determine whether A is diagonalizable. If so, give the invertible matrix P and the diagonal matrix D such that $A = PDP^{-1}$. If not, give the reason.

12. (12 pts.) Give a basis $\left\{ \begin{bmatrix} 0 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ -2 \\ 4 \end{bmatrix} \right\}$ for a subspace W, use the Gram-Schmidt process to produce an orthogonal basis for W.

13. (12 pts.) Let $\mathbf{y} = \begin{bmatrix} 6 \\ 3 \\ -2 \end{bmatrix}$, $\mathbf{u_1} = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$ and $\mathbf{u_2} = \begin{bmatrix} -4 \\ 3 \\ 0 \end{bmatrix}$.

(a) Verify that $\{\mathbf{u_1}, \mathbf{u_2}\}$ is an orthogonal set.
(b) Let $\mathbf{w}$ be the orthogonal projection of $\mathbf{y}$ onto W =Span$\{\mathbf{u_1}, \mathbf{u_2}\}$, find $\mathbf{w}$.
(c) Find the vector $\mathbf{z}$ orthogonal to W such that $\mathbf{y} = \mathbf{w} + \mathbf{z}$.
(d) Calculate $\| \mathbf{y} \|^2$, $\| \mathbf{w} \|^2$ and $\| \mathbf{z} \|^2$ and find a numerical relations among these three numbers and briefly explain why.

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