Math 20580 Name:____________________

Final Exam Instructor:____________________

May 7, 2020 Section:____________________

Calculators are NOT allowed. You will be allowed 180 minutes to do the test.

There are 20 multiple choice questions worth 7 points each. You will receive 10 points for following the instructions. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

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1.	a	b	c	d	e	11.	a	b	c	d	e
2.	a	b	c	d	e	12.	a	b	c	d	e
3.	a	b	c	d	e	13.	a	b	c	d	e
4.	a	b	c	d	e	14.	a	b	c	d	e
5.	a	b	c	d	e	15.	a	b	c	d	e
6.	a	b	c	d	e	16.	a	b	c	d	e
7.	a	b	c	d	e	17.	a	b	c	d	e
8.	a	b	c	d	e	18.	a	b	c	d	e
9.	a	b	c	d	e	19.	a	b	c	d	e
10.	a	b	c	d	e	20.	a	b	c	d	e

1. Let $\mathcal{B}$ be the basis $\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}\}$ and $\mathcal{C}$ be the basis $\{\begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix}\}$. Find the change of basis matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{\mathcal{P}}$.

(a) $\begin{bmatrix} -1 & -1/2 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 5 \\ -2 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$
(e) none of these

2. Let M be the matrix $\begin{bmatrix} -2 & 1 & -2 & 2 \\ 17 & 32 & 20 & -18 \\ -16 & -24 & -15 & 16 \\ -7 & 2 & -3 & 6 \end{bmatrix}$. You are told that M has eigenvector $\begin{bmatrix} 2 \\ 2 \\ 0 \\ 5 \end{bmatrix}$. What is the corresponding eigenvalue?

(a) 2 (b) 4 (c) 5 (d) 8 (e) none of these

3. Let $M_{6\times 8}$ be the vector space of all 6 by 8 matrices, under addition of matrices and scalar multiplication of matrices. What is the dimension of $M_{6\times 8}$?

   (a) 6 (b) 8 (c) 14 (d) 48 (e) none of these

4. Let $T : \mathbb{R}^{18} \to \mathbb{R}^{14}$ be onto. What is the dimension of the kernel of T?

   (a) 4 (b) 14 (c) 0 (d) 32 (e) not enough information to tell

5. What is the general solution of the equation $y'' - 4y' + 5y = 0$?

(a) $c_1e^{-4t} + c_2e^{5t}$ (b) $c_1e^{4t} + c_2\cos(5t)$ (c) $c_1e^{4t} + c_2e^{t}$
(d) $c_1e^{2t}\cos(t) + c_2e^{2t}\sin(t)$ (e) $c_1e^{2t} + c_2e^{-3t}$

6. Let $\mathbb{P}_2$ be the vector space of polynomials of degree at most 2, and let $\mathbb{B} = \{1, t+1, (t+1)^2\}$. With respect to $\mathbb{B}$, the coordinates of $3t^2 + 2t + 1$ are:

(a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ (e) none of the above

7. Which of the following is a subspace of the vector space of functions on the real numbers?

   (a) the set of f with $f(0) = 2$
   (b) the set of solutions to the differential equation $y'' - \sin(t)y = 0$
   (c) the set of polynomials of the form $at^3 + bt$ with $a \neq 0$
   (d) the set of solutions to the differential equation $y'' = 5$
   (e) none of these

8. Which of the following sets of vectors are linearly independent?

   (a) $\{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 11 \\ 15 \end{bmatrix}\}$ (b) $\{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}\}$ (c) $\{\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\}$

   (d) $\{\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}\}$ (e) none of these

9. What is the dimension of the null space of $A = \begin{bmatrix} 1 & 2 & 0 & 5 \\ -3 & -5 & -1 & -12 \\ 2 & 3 & 1 & 8 \\ 3 & 8 & -2 & 13 \end{bmatrix}$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) none of these

10. Suppose a 7x7 matrix A has determinant 17. Which of the following must be TRUE?

(a) the rank of A is 7 (b) $\det A^T = \dfrac{1}{17}$ (c) $\det A^{-1} = -17$
(d) $\det(A^T A) = 49$ (e) none of these

11. Consider the initial value problem $y'' - 4y' - 5y = 0$, $y(0) = 1$, $y'(0) = 0$. Which of the following describes the behavior of the solution at $t \to +\infty$:

(a) $\lim_{t\to\infty} y(t) = +\infty$ (b) $\lim_{t\to\infty} y(t) = -\infty$ (c) $\lim_{t\to\infty} y(t) = 0$
(d) $y(t)$ is a decaying oscillation (e) $y(t)$ is a growing oscillation

12. Consider the equation $y'' + t^2 y' + t^3 y = 0$. Let y_1 be the solution satisfying $y_1(0) = 1$, $y_1'(0) = 2$ and let y_2 be the solution satisfying $y_2(0) = 3$, $y_2'(0) = 4$. Using Abel's formula, find the Wronskian $W[y_1, y_2]$.
(*Hint:* fix the constant in Abel's formula by computing $W[y_1, y_2]$ at $t = 0$ directly from the initial conditions on y_1, y_2.)

(a) 0 (b) $-2e^{-t^4/4}$ (c) e^t (d) $e^{t^3/3}$ (e) $-2e^{-t^3/3}$

13. Consider the autonomous equation $y' = y(y-1)(y-2)(y-3)$ with initial condition $y(0) = 2.99$. Without solving the equation explicitly, find the limit $\lim_{t \to +\infty} y(t)$.

(a) 0 (b) 1 (c) 2 (d) 3 (e) ∞

14. Equation $x^2 y + \sin x + (\frac{x^3}{3} + e^y)y' = 0$ is:

(a) linear (b) autonomous (c) separable (d) exact
(e) none of the above

15. On which interval is the solution of the initial value problem $(\sin t)\, y'' + y = 1$, $y(1) = 1$, $y'(1) = 2$ certain to exist?

(a) $0 < t < 2\pi$ (b) $0 < t < \pi$ (c) $\pi < t < 2\pi$ (d) $-\infty < t < +\infty$
(e) cannot guarantee existence on any interval

16. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 0 & 4 \end{bmatrix}$. Find the matrix Q in the QR decomposition of A.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 3 \\ 0 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & \frac{2}{\sqrt{29}} \\ 0 & \frac{3}{\sqrt{29}} \\ 0 & \frac{4}{\sqrt{29}} \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 3/5 \\ 0 & 4/5 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix}$

(e) does not exist

17. Find the least-squares solution of the equation $A\mathbf{x} = \mathbf{b}$ with
$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

(a) $\begin{bmatrix} 2/3 \\ 2/3 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (e) does not exist

18. Find the distance between the vector $\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}$ and the subspace in $\mathbb{R}^4$ spanned by the orthogonal set $\{\mathbf{v}_1, \mathbf{v}_2\}$ with $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$.

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $\sqrt{5}$ (e) 2

19. The solution of the initial value problem $y' = \dfrac{1}{t\, e^y}$, $y(1) = 0$ is given implicitly by:

(a) $e^y = \ln t$ (b) $e^y = \ln t + 1$ (c) $\dfrac{t^2}{2} = -e^{-y} - \dfrac{1}{2}$ (d) $-e^{-y} = \ln t - 1$
(e) does not exist

20. Find the general solution of the equation $t^2y' + 4ty = 3$.

(a) $-\dfrac{3}{5}t^{-1} + Ct^4$ (b) $t^{-1} + Ce^{2t^2}$ (c) $t + Ct^4$ (d) $t^{-1} + Ct^{-4}$
(e) cannot be found explicitly using methods we learned