Name: Date: 09/16/2021

M20580 L.A. and D.E. Tutorial
Quiz 3

1. For each of the following linear transformations, determine their standard matrices:

a) $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 + x_3 \\ -x_1 + x_2 - 2x_3 \\ x_2 + 3x_3 \end{bmatrix}$ b) $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x + y \\ -2x \\ y \end{bmatrix}$

Solution: Remember that the dimension of the input is the number of columns and the dimension of the output is the number of rows. So, the first matrix is 3×3. Label the columns as x_1, x_2 and x_3 and read off the coefficients from the output:

$$\text{a)} \quad [T] = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -2 \\ 0 & 1 & 3 \end{bmatrix}$$

Similarly, in the second case there are two columns (the dimension of the input is two) and three rows:

$$\text{b)} \quad [T] = \begin{bmatrix} -1 & 1 \\ -2 & 0 \\ 0 & 1 \end{bmatrix}$$

2. For the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -1 \end{bmatrix}$, answer the following questions:

a) Does $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ belong to $\text{Col}(A)$?

b) Does $\begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$ belong to $\text{Row}(A)$?

c) Does $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ belong to $\text{Null}(A)$?

Note: we don't give credit for a guess, you should clearly show the technique or reference relevant theorems.

Solution:

a) Augment A by $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and row reduce it:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ -1 & 0 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 2 & 2 & 1 \end{array}\right]$$

We can see that the corresponding linear system is consistent, for there are no contradictions in the above REF. Therefore, the vector does belong to Col(A).

b) Augment A on the bottom by the row, transpose and row reduce:

$$\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & -1 & -1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 0 & 2 & -4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 0 & 0 & -3 \end{array}\right].$$

There is a contradiction in the last row: $0 = -3$, which indicates the system is not consistent. Therefore, the row does not belong to Row(A).

c) To answer this question, multiply A by the given vector:

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Since the resulting vector is the zero vector, we conclude that the vector belongs to Null(A).