Name: Date: 10/14/2021

**M20580 L.A. and D.E.**
**Quiz 6**

1. Let $V = M_{2\times 3}$ be the vector space of $2 \times 3$ matrices and consider the subspace

$$W = \left\{ \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \mid a + b + c = 0 \text{ and } d = 0 \right\}.$$

What is the dimension of $W$?

***Solution:*** Substituting the relations $a = -b - c$ and $d = 0$ into a typical element of $M_{2\times 3}$, we see that a typical element of $W$ is given by

$$\begin{bmatrix} -b-c & b & c \\ 0 & e & f \end{bmatrix} = b \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + c \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + e \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} + f \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Now the four matrices appearing on the right hand side of the above equation are easily seen to be linearly independent, and so they are a basis for $W$, which therefore then has dimension 4.

2. If $\mathcal{B} = \{1 + t^2, t^2, -4t\}$ is a basis for $\mathcal{P}_2$, then what is $p(t)$ if $[p(t)]_{\mathcal{B}} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$?

***Solution:*** We know from $[p(t)]_{\mathcal{B}} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$ that

$$p(t) = 1(1 + t^2) - 2(t^2) + 0(-4t) = 1 + t^2 - 2t^2 = 1 - t^2.$$