Name: Date: 11/11/2021

M20580 L.A. and D.E.
Quiz 8

1. Find the least squares solution to the following system:

$$\begin{cases} x_1 + x_2 &= 1 \\ x_1 - x_2 &= 1 \\ x_1 &= -1 \end{cases}$$

> ***Solution:*** The matrix of the system is
>
> $$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix}.$$
>
> From $A^T A\mathbf{x} = A^t\mathbf{x}$, we have a system
>
> $$\begin{cases} 3x_1 = 1 \\ 2x_2 = 0 \end{cases}$$
>
> hence the least squares solution is given by $x_1 = 1/3$ and $x_2 = 0$. Note that despite the fact $x_1 = -1$ was given in third initial equation, the least squares procedure assigned a different value to x_1 to reconcile it in an optimal way with the other two equations.

2. Find a basis for the orthogonal complement of a subspace V of $\mathbb{R}^3$ given by

$$V = \text{span}\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix} \}.$$

> ***Solution:*** If A is a matrix obtained by concatenating basis columns of V, then $V^\perp = \text{null}(A^T)$. So,
>
> $$A^T = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 3 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \end{bmatrix}.$$
>
> From the last RREF, we see that
>
> $$V^\perp = \text{null}(A^T) = \text{span}\{ \begin{bmatrix} -4 \\ -2 \\ 1 \end{bmatrix} \}.$$