Name: Date: 12/2/2021

M20580 L.A. and D.E.
Quiz 9

1. Find the general solution of the following differential equation:

$$y'' + 4y' + 4y = 0.$$

Solution: Let's solve the auxiliary quadratic equation:

$$m^2 + 4m + 4 = (m+2)^2 = 0 \quad \Rightarrow \quad m = -2;$$

therefore, we can choose e^{-2x} and xe^{-2x} to be a pair of linearly independent solutions. Thus, the general solution is

$$y(x) = c_1 e^{-2x} + c_2 x e^{-2x}.$$

2. A tank originally has 10 liters of brine with a concentration of 1 gram of salt per liter. Brine with concentration of 2 grams of salt per liter is pumped into the tank at a rate of 3 liters per second. The mixture is kept stirred and is pumped out at a rate of 2 liters per second. Find a differential equation for the amount of salt(in terms of time). You are **NOT** required to solve the differential equation you get.

Solution: Let $y(t)$ be the amount of salt in the tank at t minutes, then

$$\frac{dy}{dt} = \text{rate of incoming salt} - \text{rate of outgoing salt}.$$

and

$$\begin{aligned}
\text{rate of incoming salt} &= (\text{rate of incoming volume of brine}) \times (\text{incoming density}) \\
&= 3 \times 2 = 6 \\
\text{rate of outgoing salt} &= (\text{rate of outgoing volume of brine}) \times (\text{outgoing density}) \\
&= 2 \times \frac{y(t)}{10 + (3-2)t} = \frac{2y(t)}{10+t}.
\end{aligned}$$

so we obtain the 1st linear order DE

$$\frac{dy}{dt} = 6 - \frac{2}{10+t}y.$$