Math 20580 Tutorial – Quiz 1

1. Determine whether the vector $\mathbf{w}$ can be written as a linear combination of the vectors $\mathbf{v}_1, \mathbf{v_2}$ and $\mathbf{v}_3$. If yes, find scalars a_1, a_2, a_3 such that $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 = \mathbf{w}$.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 6 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 3 \\ 4 \\ -9 \end{bmatrix} \text{ and } \mathbf{w} = \begin{bmatrix} 3 \\ -7 \\ -4 \end{bmatrix}.$$

> **Solution:** To solve $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 = \mathbf{w}$, row reduce to the corresponding augmented matrix
>
> $$\begin{bmatrix} 1 & -2 & 3 & 3 \\ 0 & 1 & 4 & -7 \\ -3 & 6 & -9 & -4 \end{bmatrix} \xrightarrow{R_3+3R_1} \begin{bmatrix} 1 & -2 & 3 & 3 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$
>
> By the third row, $0a_1 + 0a_2 + 0a_3 = 5$, which is impossible. Thus $\mathbf{w}$ cannot be written as a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$.

2. Determine the general solution to the system of equations below.

$$8x_1 + 5x_2 = 5$$
$$2x_1 + x_2 = 3.$$

> **Solution:** To solve the system of equations, set up an augmented matrix, and row reduce.
>
> $$\begin{bmatrix} 8 & 5 & 5 \\ 2 & 1 & 3 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 2 & 1 & 3 \\ 8 & 5 & 5 \end{bmatrix} \xrightarrow{R_2-4R_1} \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & -7 \end{bmatrix} \xrightarrow{R_1-R_2} \begin{bmatrix} 2 & 0 & 10 \\ 0 & 1 & -7 \end{bmatrix}$$
>
> From this, we see $x_2 = -7$ and $x_1 = 5$.