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M20580 L.A. and D.E.
Quiz 10

1. The solution of the initial value problem

$$y' = \frac{1}{te^y}, \quad y(1) = 0$$

is given implicitly by

a. $e^y = \ln t$

b. $e^y = \ln t + 1$

c. $\frac{t^2}{2} = -e^{-y} - \frac{1}{2}$

d. $-e^{-y} = \ln t - 1$

e. does not exist

Solution: This ODE is seperable. We have

$$e^y dy = \frac{1}{t} dt.$$

Integrate on both sides, we get $e^y = ln(t) + C$ and then we plug in the initial condition to get $C = 1$. Thus b is correct.

2. Find the general solution of the equation $t^2 y' + 4ty = 3$.

a. $-\frac{3}{5}t^{-1} + Ct^4$

b. $t^{-1} + Ce^{2t^2}$

c. $t + Ct^4$

d. $t^{-1} + Ct^{-4}$

e. cannot be found explicitly using methods we learned

Solution: The answer is d. We first turn it into standard form:

$$y' + \frac{4}{t}y = \frac{3}{t^2}.$$

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The integrating factor is $e^{\int 4/t dt} = e^{4 \ln t} = t^4$. Then

$$\begin{aligned} y(t) &= \frac{\int t^4 3/t^2 dt}{t^4} \\ &= \frac{\int 3t^2 dt}{t^4} \\ &= \frac{t^3 + C}{t^4} \\ &= t^{-1} + Ct^{-4} \end{aligned}$$