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Math 20580 Tutorial
Quiz 2

1. Let $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 10 & -5 \end{bmatrix}$. Determine whether the matrix is invertible and find its inverse if it exists.

> ***Solution:*** Recall that a 2×2 matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible if and only if $\det M = ad - bc \neq 0$. The inverse matrix is given by
>
> $$M^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$
>
> Since $\det A = 8 - 9 = -1 \neq 0 (1pt)$, then matrix A is invertible and $A^{-1} = -\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix} (2pt)$.
>
> For matrix B, $\det B = 10 - 10 = 0 (1pt)$. Thus, matrix B is not invertible$(1pt)$.

2. Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be a linear transformation defined by

$$T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_2 + x_3 \\ 2x_1 + x_3 \end{bmatrix}, \text{for any } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^{\mathbf{3}}.$$

Find a matrix A such that $T(\mathbf{x}) = A\mathbf{x}$.

> ***Solution:*** $A = \begin{bmatrix} T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & T \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} & T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} (3pt) = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} (2pt).$
>
> If students set $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$, a 2×3 matrix, and solve for the variables, they are still on the right track(3pt).