Name: Date: 09/14/2023

Math 20580 L.A. and D.E. Tutorial
Quiz 3

CALCULATORS ARE NOT ALLOWED.

1. True or False: Given three **linearly independent** vectors $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$, then any vector in $\text{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ is in the $\text{Span}(\mathbf{v}_1, \mathbf{v}_2)$.

> ***Solution:*** 2 points. This is false. Since we assumed the vectors are linearly independent then $\mathbf{v}_3$ cannot be in $\text{Span}(\mathbf{v}_1, \mathbf{v}_2)$.

2. Consider the matrix:

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$$

   (a) Find a basis for $\text{Col}(A)$.

   (b) Is the vector $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ in $\text{Col}(A)$?

   (c) Is the vector $\begin{bmatrix} 1 & 3 & 4 \end{bmatrix}$ in $\text{Row}(A)$?

   (d) Is the vector $\begin{bmatrix} 1 \\ 1 \\ -5 \end{bmatrix}$ in $\text{Null}(A)$?

> ***Solution:*** 8 points total, 2 points for each part. Putting A in row echelon form gives:
>
> $$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 5 & 1 \end{bmatrix}.$$
>
> Thus a basis for $\text{Col}(A)$ come from the first 2 columns for A, i.e the vectors $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$.
>
> For (b) we wish to figure out if there are a, b in $\mathbb{R}$ such that
>
> $$a\begin{bmatrix} 1 \\ 2 \end{bmatrix} + b\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$
>
> This gives rise to a system of linear equations, and the corresponding augmented matrix can be put in REF as follows:
>
> $$\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 2 & 3 & 4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 5 & 2 \end{array}\right].$$

Thus we see this has a unique solution and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ is in the column space.

Similarly for (c) we wish to know if

$$a \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} + b \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 4 \end{bmatrix}$$

. This gives us a system of equations

$$\begin{cases} a + 2b = 1 \\ -a + 3b = 3 \\ b = 4 \end{cases}$$

whose corresponding augmented matrix can be put in REF as follows:

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ -1 & 3 & 3 \\ 0 & 1 & 4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 5 & 4 \\ 0 & 1 & 4 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & -16 \\ 0 & 1 & 4 \end{array}\right]$$

So the system is inconsistent and the vector is not in the row space.

For (d) we have

$$A \begin{bmatrix} 1 \\ 1 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 - 1 + 0 \\ 2 + 3 - 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and thus the vector is in the null space.