Name: Date: 09/28/2023

M20580 L.A. and D.E. Tutorial
Quiz 4

1. Recall that $\mathcal{P}_n$ denotes the vector space of degree *at most* n. Which of the following is NOT a linear transformation? (*Hint:* There is only one correct answer)

   A. $T : \mathcal{P}_3 \to \mathcal{P}_2$, where $T(p(t)) = p'(t)$,
   B. $T : \mathcal{P}_3 \to \mathcal{P}_3$, where $T(p(t)) = tp'(t) - 3t^2$,
   C. $T : \mathcal{P}_3 \to \mathbb{R}$, where $T(p(t)) = p'(2)$,
   D. $T : \mathcal{P}_3 \to \mathbb{R}$, where $T(p(t)) = p(0)$,
   E. All of them are linear transformations.

Solution: (2pt) Choice B is not a linear transformation, because it maps zero polynomial to $-3t^2$.

2. Let $p(x) = 1 - 2x$, $q(x) = x - x^2$, and $r(x) = -2 + 3x + x^2$ be polynomials in $\mathcal{P}_2$. Determine whether $s(x) = 3 - x - 5x^2$ is in span $\{p(x), q(x), r(x)\}$.

Solution: The coordinate vectors of these polynomials with respect to the standard basis of $\mathcal{P}_2$ are(4pt)

$$[p(x)]_{std} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \quad [q(x)]_{std} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \quad [r(x)]_{std} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \quad [s(x)]_{std} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}.$$

The equation $ap(x) + bq(x) + cr(x) = s(x)$ in the unknown a, b, c, gives us a linear system whose augmented matrix is(1pt)

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ -2 & 1 & 3 & -1 \\ 0 & -1 & 1 & -5 \end{array}\right].$$

This row reduces to(2pt)

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right],$$

which tell us there are infinitely many solutions. So, $s(x)$ is in span $\{p(x), q(x), r(x)\}$(1pt).