Name: Date: 10/05/2021

M20580 L.A. and D.E. Tutorial
Quiz 5

1. A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ has out puts

$$T\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}2\\4\end{bmatrix}, \qquad T\begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}1\\3\end{bmatrix}.$$

Find $T\begin{bmatrix}3\\1\end{bmatrix}$.

Solution: The augmented matrix $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 3 & 2 & 1 \end{array}\right]$ has REF$\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right]$.Thus,

$$\begin{bmatrix}3\\1\end{bmatrix} = -\begin{bmatrix}1\\3\end{bmatrix} + 2\begin{bmatrix}2\\2\end{bmatrix}.$$

Thus

$$T\begin{bmatrix}3\\1\end{bmatrix} = -T\begin{bmatrix}1\\3\end{bmatrix} + 2T\begin{bmatrix}2\\2\end{bmatrix} = -\begin{bmatrix}2\\4\end{bmatrix} + 2\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}0\\2\end{bmatrix}.$$

2. Let $B = \{1 - t + t^2,\ t - t^2,\ t + t^2\}$ be a basis for the space $\mathcal{P}_2$ of polynomials of degree at most 2. Find the coordinate vector $[p]_B$ of $p(t) = 2 + t + 3t^2$.

Solution: We have that the augmented matrix in terms of the standard basis $\{1, t, t^2\}$ is

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 3 \end{array}\right].$$

Putting in RREF gives

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right].$$

Hence $[p]_B = \begin{bmatrix}2\\1\\2\end{bmatrix}$, i.e. $p(t) = 2(1 - t + t^2) + 1(t - t^2) + 2(t + t^2)$.