Name: Date: 10/12/2023

MATH 20580 L.A. and D.E. Tutorial
Quiz 6

CALCULATORS ARE NOT ALLOWED

1. Find the determinant of the following matrix $A = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 2 & 2 & 1 & 4 \\ 0 & 0 & -1 & 1 \\ 1 & 3 & 0 & -3 \end{pmatrix}$. **Hint**: find the row/column with most 0's first.

   A. -5 B. -3 C. 5 D. 3 E. 0

Solution: Laplace expansion along the 3rd column gives $\det(A) = (0)C_{13} + (1)C_{23} + (-1)C_{33} + (0)C_{43}$.

Now computing the cofactors we have

$$C_{23} = (-1)^{2+3}M_{23} = -M_{23} = -\det \begin{pmatrix} 1 & 2 & -1 \\ 0 & 0 & 1 \\ 1 & 3 & -3 \end{pmatrix} = (-1)(-1(3-2)) = 1$$

Likewise, the other nonzero cofactor C_{33} is computed

$$C_{33} = (-1)^{3+3}M_{33} = M_{33} = \det \begin{pmatrix} 1 & 2 & -1 \\ 2 & 2 & 4 \\ 1 & 3 & -3 \end{pmatrix}$$

$= 1(2(-3)-(3)(4))-(2)(2(-3)-(4)(1))+(-1)((2)(3)-(2)(1)) = -18+20-4 = -2$

And so we have $\det(A) = +(1)(1) + (-1)(-2) = 3$. Answer choice D.

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2. Use Cramer's rule to solve the system

$$\begin{bmatrix} 1 & -1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

(a) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0.5 \end{bmatrix}$ **(b)** $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix}$ **(c)** $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ **(e)** $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1.5 \\ -0.5 \end{bmatrix}$

Solution: Answer choice (b) is correct.

The determinant of the matrix is 2. Replacing the first column with the solution vector we obtain the matrix $\begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix}$ which has determinant 3. Hence $x_1 = 3/2$.

Replacing the second column with the solution vector we obtain the matrix $\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$ which has determinant 1. Hence $x_2 = 1/2$.