Name: Date: 11/02/2023

Math 20580 L.A. and D.E. Tutorial
Quiz 7

CALCULATORS ARE NOT ALLOWED

1. Which of the following subset of $\mathbb{R}^3$ is an **orthonormal** set?

   A. $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}\right\}$ B. $\left\{\begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}\right\}$ C. $\left\{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\end{bmatrix}, \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\-1\end{bmatrix}\right\}$

   D. $\left\{\frac{1}{\sqrt{3}}\begin{bmatrix}2\\0\\1\end{bmatrix}, \frac{1}{\sqrt{3}}\begin{bmatrix}1\\0\\-2\end{bmatrix}\right\}$ E. $\left\{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\\0\end{bmatrix}\right\}$.

   Solution: We need to check for each choice that the dot product of the two vectors is 0 (orthogonality) and that the norm (length) of each vector in the set is 1 (normalization).

   The sets in Choices A and Choice E are not even orthogonal sets. The ones in Choices B and D are orthogonal sets but not orthonormal since they contain a vector whose norm is not 1.

   Only the set in **Choice C** satisfies both conditions so it is the answer.

2. What is projection $\text{proj}_W\mathbf{v}$ of the vector $\mathbf{v} = \begin{bmatrix}1\\-1\\2\end{bmatrix}$ on the plane $W = \text{span}\left\{\begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}-2\\0\\1\end{bmatrix}\right\}$?

   (Hint: To save your calculation, look for which pairs of vectors that are already orthogonal).

   A. $\begin{bmatrix}3\\6\\6\end{bmatrix}$ B. $\begin{bmatrix}0\\0\\0\end{bmatrix}$ C. $\frac{1}{3}\begin{bmatrix}1\\-1\\2\end{bmatrix}$ D. $\frac{1}{3}\begin{bmatrix}1\\2\\2\end{bmatrix}$ E. $-\frac{1}{3}\begin{bmatrix}1\\2\\2\end{bmatrix}$

   Solution: Answer choice D is correct.

   Let $\mathbf{v}_1 = \begin{bmatrix}1\\2\\2\end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix}-2\\0\\1\end{bmatrix}$. Then $W = \text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$ and note that $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$ so we can use the orthogonal projection formula. Also $\mathbf{v} \cdot \mathbf{v}_2 = 0$, so $\mathbf{proj}_{\mathbf{v}_2}\mathbf{v} = \mathbf{0}$. Hence,

   $$\mathbf{proj}_W\mathbf{v} = \mathbf{proj}_{\mathbf{v}_1}\mathbf{v} + \mathbf{proj}_{\mathbf{v}_2}\mathbf{v} = \frac{\mathbf{v}\cdot\mathbf{v}_1}{\mathbf{v}_1\cdot\mathbf{v}_1}\mathbf{v}_1 + \mathbf{0} = \frac{1\cdot 1 + 2\cdot(-1) + 2\cdot 2}{1^2+2^2+2^2}\begin{bmatrix}1\\2\\2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1\\2\\2\end{bmatrix}.$$