Name: Date: 11/09/2023

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Quiz 8

1. Find an orthonormal basis for
$$\operatorname{span}\left\{x_1 = \begin{bmatrix}1\\2\end{bmatrix}, x_2 = \begin{bmatrix}3\\1\end{bmatrix}\right\}.$$

Solution: Apply Gram-Schmidt. Our first basis element is
$$e_1 = \frac{1}{\sqrt{1+4}}\begin{bmatrix}1\\2\end{bmatrix} = \frac{1}{\sqrt{5}}\begin{bmatrix}1\\2\end{bmatrix}.$$
Next note that $e_1 \cdot x_2 = \frac{1}{\sqrt{5}}(3+2) = 5/\sqrt{5} = \sqrt{5}$ and thus the second basis element is
$$v_2 = x_2 - (e_1 \cdot x_2)e_1 = x_2 - \sqrt{5}e_1 = \begin{bmatrix}3\\1\end{bmatrix} - \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}2\\-1\end{bmatrix}.$$
Normalizing gives
$$e_2 = \frac{1}{\sqrt{5}}\begin{bmatrix}2\\-1\end{bmatrix}.$$
So $\{e_1, e_2\}$ give an orthonormal basis for $\operatorname{span}\{x_1, x_2\}$.

2. Find a basis for the orthogonal complement of a subspace V of $\mathbb{R}^3$ given by
$$V = \operatorname{span}\left\{\begin{bmatrix}1\\0\\3\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix}\right\}.$$

Solution: If A is a matrix obtained by concatenating basis columns of V, then $V^{\perp} = \operatorname{null}(A^T)$. So,
$$A^T = \begin{bmatrix}1 & 0 & 3\\2 & 1 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 3\\0 & 1 & -6\end{bmatrix}.$$
From the last RREF, we see that
$$V^{\perp} = \operatorname{null}(A^T) = \operatorname{span}\left\{\begin{bmatrix}-3\\6\\1\end{bmatrix}\right\}.$$