M20580 L.A. and D.E. Tutorial
Quiz 9

CALCULATORS ARE NOT ALLOWED.

1. Consider the following differential equation:

$$\frac{d^2u}{dr^2} + r^2\frac{du}{dr} + u = \ln(r)$$

Which of the following statements are true?

  I The differential equation is second order.

  II The differential equation is linear.

(a) Neither statement is true   (b) Only I is true   (c) Only II is true
(d) I and II are both true   (e) Not enough information

Solution: Answer choice **(d)**. The highest-order derivative appearing is $\frac{d^2u}{dr^2}$ in the first term, so the equation is 2nd-order, i.e. I holds. The differential equation is of the form

$$f_2(r)\frac{d^2u}{dr^2} + f_1(r)\frac{du}{dr} + f_0(r)u = g(r)$$

for some functions $f_i(r)$ and $g(r)$, and is thus linear, i.e. II holds.

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2. Which of the following is an integrating factor of the differential equation?

$$x^2y' - 2y = x\ln(x), \quad (x > 0).$$

(a) e^{-2x} (b) $e^{2/x}$ (c) $e^{\ln x}$ (d) $\frac{\ln(x)}{x}$ (e) $-2/x$

Solution: Dividing by x^2, the standard form of the given DE is

$$y' - \frac{2}{x^2}y = x\ln(x).$$

From this form we identify $P(x) = -2/x^2$. Hence the integrating factor is

$$e^{-\int \frac{2}{x^2}\,dx} = e^{2/x}.$$

Thus the answer is **(b)**.