Math 20580 Name:__________________________

Midterm 1 Instructor:__________________________

February 13, 2020 Section:__________________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are finished.

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

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1. a b c d e
2. a b c d e
3. a b c d e
4. a b c d e
5. a b c d e
6. a b c d e
7. a b c d e
8. a b c d e

Multiple Choice.

9.

10.

11.

12.

Total.

Part I: Multiple choice questions (7 points each)

1. What can be said about the following system of linear equations?

$$\begin{cases} 2x_1 - 4x_3 = 5 \\ x_2 - 3x_3 = 3 \end{cases}$$

(a) The solution set is a subspace of $\mathbb{R}^3$ (b) The system is inconsistent

(c) There are only finitely many solutions (d) Every solution is in $\mathbb{R}^2$

(e) none of the above

2. What are the values of h and k for which the matrix below is not invertible?

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & k \\ -1 & h & -1 \end{bmatrix}$$

(a) $h = 0$ and $k = 0$ (b) $h = -1$ or $k = -1$

(c) $h = 1$ and any k (d) $h = 0$ and $k = 1$

(e) none of the above

3. Under which of the scenarios below does the equation $A\vec{x} = \vec{0}$ have a nontrivial solution.

   1. A is a 3×3 matrix with three pivot positions.
   2. A is a 4×4 matrix with two pivot positions.
   3. A is a 2×5 matrix with two pivot positions.
   4. A is a 5×3 matrix with three pivot positions.

   (a) 2 only (b) 2,3 only (c) 1,4 only (d) 2,3,4 only (e) 4 only

4. If the matrices A, B are such that

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \qquad AB = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 5 & 2 \end{bmatrix},$$

   then what is the matrix B?

   (a) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 2 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (e) It can't be determined.

5. Suppose that a 5×6 matrix A has a nullspace of dimension 2. How many rows of zeros does the reduced echelon form of A contain?

(a) 3 (b) 2 (c) 5 (d) 4 (e) 1

6. Which of the following matrices has linearly independent columns?

$$A = \begin{bmatrix} 3 & -4 \\ 4 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -1 \\ 2 & 0 \\ 0 & 1 \\ -2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & -3 \\ 4 & -5 \\ 5 & -6 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 2 & 4 \\ 0 & 3 & 6 \\ 0 & 5 & 10 \end{bmatrix}$$

(a) A only (b) A,B only (c) A,B,C only (d) D only (e) B, C only

7. Consider a basis $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ of $\mathbb{R}^3$, and a linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ with the property that

$$T(\vec{b}_1) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \qquad T(\vec{b}_2) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \qquad T(\vec{b}_3) = \begin{bmatrix} 0 \\ 3 \end{bmatrix}.$$

If $\vec{u}$ has coordinate vector $[\vec{u}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ relative to $\mathcal{B}$, then $T(\vec{u})$ is equal to

(a) $\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ (b) $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$ (c) $\vec{b}_1 - \vec{b}_2 + 2\vec{b}_3$ (d) $\begin{bmatrix} 0 \\ 6 \end{bmatrix}$ (e) $\begin{bmatrix} -2 \\ -1 \end{bmatrix}$

8. The rank of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 & 3 & 1 \\ 6 & -3 & 3 & 3 & 1 \\ 4 & -2 & 2 & 0 & 0 \end{bmatrix}$$

is

(a) 1 (b) 3 (c) 4 (d) 2 (e) 0

Part II: Partial credit questions (11 points each). Show your work.

9. (a) Find the standard matrix for each of the following transformations $\mathbb{R}^2 \to \mathbb{R}^2$:

- Counterclockwise rotation with angle $\pi/4$.
- Projection to the y-axis.

(b) Find the standard matrix of the linear transformation that consists of counterclockwise rotation with angle $\pi/4$, followed by projection to the y-axis.

(c) Find the standard matrix of the linear transformation that consists of projection to the y-axis, followed by counterclockwise rotation with angle $\pi/4$.

10. Find the inverse of the matrix

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

11. Consider the matrix

$$A = \begin{bmatrix} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find the rank of A, and find a basis $\mathcal{B}$ for $\mathrm{Col}(A)$.

(b) If $\vec{a}_5$ is the fifth column of A, determine its coordinate vector $[\vec{a}_5]_{\mathcal{B}}$ relative to $\mathcal{B}$.

12. Consider the linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ given by

$$T(x_1, x_2) = \begin{bmatrix} x_1 - 2x_2 \\ x_1 + 2x_2 \\ 2x_1 - 3x_2 \end{bmatrix}.$$

(a) Find the standard matrix of T.

(b) Explain why every vector $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ in the range of T satisfies $7b_1 + b_2 - 4b_3 = 0$.

(c) Write down two distinct vectors $\vec{v}_1, \vec{v}_2$ that are not contained in the range of T (and make sure to explain why they are not contained in the the range).