Math 20580 Name:________________________
Midterm 3 Instructor:________________________
November 16, 2021 Section:________________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 75 minutes to do the test. You may leave earlier if you are finished.

There are 8 multiple choice questions worth 7 points each and 4 partial credit questions each worth 11 points. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

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1. a b c d e
2. a b c d e
3. a b c d e
4. a b c d e
5. a b c d e
6. a b c d e
7. a b c d e
8. a b c d e

Multiple Choice.

9.

10.

11.

12.

Total.

Part I: Multiple choice questions (7 points each)

1. Consider the line L spanned by the vector $\vec{v} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$. The distance from the vector $\vec{x} = \begin{bmatrix} -7 \\ 1 \end{bmatrix}$ to the line L is

(a) $\sqrt{5}$ (b) $\sqrt{45}$ (c) $2\sqrt{3}$ (d) 5 (e) $\sqrt{50}$

2. Consider the matrices

$$A = \begin{bmatrix} 2 & 4 & -2 & 1 & 11 \\ 3 & 6 & -3 & 1 & 15 \\ -1 & -2 & 1 & 2 & 2 \\ 4 & 8 & -4 & 4 & 28 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where B is the reduced row echelon form of A. A basis for the orthogonal complement of the row space of A is given by

(a) $\left\{ \begin{bmatrix} 2 \\ 3 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 4 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ -3 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ -4 \\ 4 \\ 28 \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} 4 \\ 6 \\ -2 \\ 8 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 11 \\ 15 \\ 2 \\ 28 \end{bmatrix} \right\}$

3. Consider the line L spanned by the unit vector $\vec{u} = \begin{bmatrix} 3/5 \\ -4/5 \end{bmatrix}$, and let $\text{proj}_L : \mathbb{R}^2 \to \mathbb{R}^2$ denote the linear transformation that sends a vector to its orthogonal projection onto the line L. The standard matrix of the transformation proj_L is

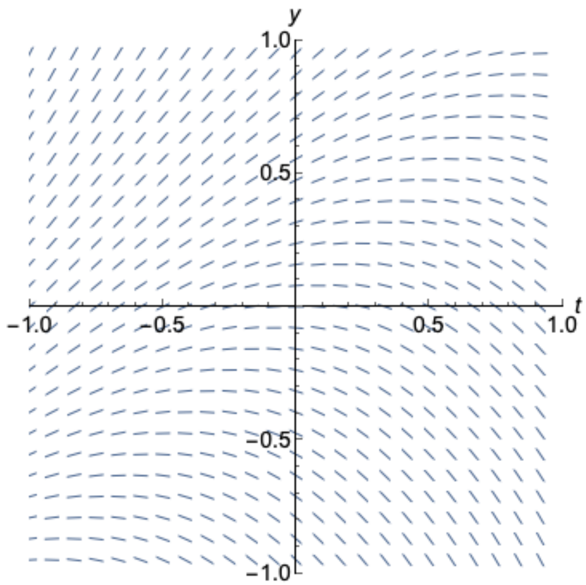
(a) $\begin{bmatrix} 3 & 0 \\ 0 & -4 \end{bmatrix}$ (b) $\frac{1}{5}\begin{bmatrix} 3 & 5 \\ 5 & -4 \end{bmatrix}$ (c) $\frac{1}{25}\begin{bmatrix} 9 & -12 \\ -12 & 16 \end{bmatrix}$ (d) $\frac{1}{5}\begin{bmatrix} 3 & 0 \\ 0 & -4 \end{bmatrix}$

(e) none of the above

4. Which of the following functions is the solution of the equation $y' = ty$ with $y(0) = 1$?

(a) t (b) $e^{\cos t}$ (c) $t^2/2$ (d) e^t (e) $e^{t^2/2}$

5. Determine $f(t, y)$ if the differential equation $\frac{dy}{dt} = f(t, y)$ has direction field (the value of t is measured on the horizontal axis, and the value of y on the vertical axis)



(a) $\sin(y)$ (b) $t^2 + y^2$ (c) y (d) $-t$ (e) $y - t$

6. Consider the autonomous equation $y' = y(y-1)(y-2)(y-3)$ with initial condition $y(0) = 2.99$. Without solving the equation explicitly, find the limit $\lim_{t \to +\infty} y(t)$.

(a) 0 (b) 1 (c) 2 (d) 3 (e) ∞

7. The differential equation

$$\frac{dy}{dt} + ty^2 = 0$$

is

(a) an equation of order 2 (b) a partial differential equation (c) linear
(d) separable (e) none of the above

8. Which of the following sets of vectors are orthogonal?

(I) $\left\{ \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \\ -7 \end{bmatrix} \right\}$ (II) $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix} \right\}$

(III) $\left\{ \begin{bmatrix} 2 \\ -7 \\ -1 \end{bmatrix}, \begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} \right\}$ (IV) $\left\{ \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix} \right\}$

(a) I only (b) I and III only (c) II only (d) II and IV only (e) none of these

Part II: Partial credit questions (11 points each). Show your work.

9. Let $W = \text{Span}\{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$, where

$$\vec{w}_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 1 \end{bmatrix}, \quad \vec{w}_2 = \begin{bmatrix} 1 \\ 3 \\ -1 \\ -1 \end{bmatrix}, \quad \vec{w}_3 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ -3 \end{bmatrix}.$$

(a) Apply the Gram-Schmidt process to find an orthonormal basis for W.

(b) Find the QR decomposition of the matrix A with columns $\vec{w}_1, \vec{w}_2, \vec{w}_3$.

10. Let $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ -2 & 0 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$.

(a) Find the least squares solution to the equation $A\vec{x} = \vec{b}$.

(b) Find the vector in the column space of A which is closest to $\vec{b}$.

11. Consider the differential equation

$$y + (2x - 4y^2) \cdot \frac{dy}{dx} = 0.$$

(a) Explain why the equation is not exact.

(b) Find an integrating factor μ which only depends on the variable y.

(c) Write down the implicit solution which satisfies the initial condition $y(1) = 1$.

12. (a) Find the solution of the initial value problem

$$\begin{cases} t^2 y' + 4ty = 3 \\ y(1) = -1 \end{cases}$$

(b) Find the maximal interval on which the solution to the initial value problem above is defined.