Math 20580
Final Exam
December 12, 2017

Name:______________________________
Instructor:______________________________
Section:______________________________

Calculators are NOT allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 2 hours to do the test. You may leave earlier if you are finished.

There are 20 multiple choice questions worth 7 points each. You will receive 10 points for being present and following the instructions. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Sign the pledge. "On my honor, I have neither given nor received unauthorized aid on this Exam":

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1.	a	b	c	~~d~~	e	11.	a	b	~~c~~	d	e
2.	a	~~b~~	c	d	e	12.	a	b	c	~~d~~	e
3.	~~a~~	b	c	d	e	13.	a	~~b~~	c	d	e
4.	a	b	c	d	~~e~~	14.	a	b	c	d	~~e~~
5.	a	b	~~c~~	d	e	15.	a	b	c	~~d~~	e
6.	a	~~b~~	c	d	e	16.	~~a~~	b	c	d	e
7.	~~a~~	b	c	d	e	17.	~~a~~	b	c	d	e
8.	a	~~b~~	c	d	e	18.	a	b	c	d	~~e~~
9.	a	b	~~c~~	d	e	19.	a	~~b~~	c	d	e
10.	a	b	c	d	~~e~~	20.	a	b	~~c~~	d	e

1. Find $T\begin{bmatrix}1\\1\end{bmatrix}$ if $T:\mathbb{R}^2\to\mathbb{R}^3$ is a linear transformation such that

$$T\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}2\\0\\6\end{bmatrix}\quad\text{and}\quad T\begin{bmatrix}3\\4\end{bmatrix}=\begin{bmatrix}0\\-2\\4\end{bmatrix}.$$

(a) $\begin{bmatrix}2\\2\\2\end{bmatrix}$ (b) $\begin{bmatrix}2\\-2\\10\end{bmatrix}$ (c) $\begin{bmatrix}1\\-1\\5\end{bmatrix}$ (d) $\begin{bmatrix}-1\\-1\\-1\end{bmatrix}$

(e) cannot be determined from the given information.

$$\begin{bmatrix}1\\1\end{bmatrix}=-\frac{1}{2}\cdot\begin{bmatrix}1\\2\end{bmatrix}+\frac{1}{2}\cdot\begin{bmatrix}3\\4\end{bmatrix}$$

$$\Rightarrow T\begin{bmatrix}1\\1\end{bmatrix}=-\frac{1}{2}\cdot T\begin{bmatrix}1\\2\end{bmatrix}+\frac{1}{2}\cdot T\begin{bmatrix}3\\4\end{bmatrix}$$

$$=-\frac{1}{2}\cdot\begin{bmatrix}2\\0\\6\end{bmatrix}+\frac{1}{2}\cdot\begin{bmatrix}0\\-2\\4\end{bmatrix}=\begin{bmatrix}-1\\0\\-3\end{bmatrix}+\begin{bmatrix}0\\-1\\2\end{bmatrix}=\begin{bmatrix}-1\\-1\\-1\end{bmatrix}$$

2. Find the solution of the initial value problem

$$\begin{cases}4y''-4y'+y=0\\ y(2)=4e,\quad y'(2)=3e\end{cases}$$

(a) $4e^{2t-3}$ (b) $(t+2)e^{t/2}$ (c) $4e^{t/2}+t/2-1$ (d) $5e^{t/2}-e^{-t/2}$ (e) $e\cdot t^2$

$4r^2-4r+1=0$

$r_1=r_2=\frac{1}{2}$

$y=c_1e^{t/2}+c_2\cdot te^{t/2}$

$y'=\frac{c_1}{2}e^{t/2}+c_2e^{t/2}+\frac{c_2}{2}te^{t/2}$

$y(2)=c_1e+2c_2e=4e$

$y'(2)=\frac{c_1}{2}e+2c_2\cdot e=3e$

subtract

$\frac{c_1}{2}\cdot e=e \Rightarrow \boxed{c_1=2}$

$c_1+2c_2=4\Rightarrow \boxed{c_2=1}$

$\boxed{y=2e^{t/2}+te^{t/2}=(t+2)e^{t/2}}$

3. If $\begin{bmatrix}1\\2\\1\end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix}4 & 0 & -1\\3 & 0 & 3\\2 & -2 & 5\end{bmatrix}$ then the corresponding eigenvalue is

(a) 3 (b) 1 (c) -1 (d) -3 (e) 0

$$\begin{bmatrix}4 & 0 & -1\\3 & 0 & 3\\2 & -2 & 5\end{bmatrix}\begin{bmatrix}1\\2\\1\end{bmatrix} = \begin{bmatrix}3\\6\\3\end{bmatrix} = 3\begin{bmatrix}1\\2\\1\end{bmatrix}$$

4. Find the integrating factor that would make the following equation exact:

$$\underbrace{y^2 + \sin x}_{M} + \underbrace{xy}_{N}\frac{dy}{dx} = 0$$

(a) $\mu = e^{xy^2/2}$ (b) $\mu = e^{y^2}$ (c) $\mu = \frac{x^2y^2}{2}$ (d) $\mu = y\sin(x)$ (e) $\mu = x$

$M_y = 2y$

$N_x = y$

$$\frac{M_y - N_x}{y} = \frac{2y - y}{y} = 1$$

only depends on x

$$\frac{d\mu}{dx} = \frac{M_y - N_x}{y} = 1 \rightsquigarrow \boxed{\mu = x}$$

5. The eigenvalues of the matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ are

(a) $-2 \pm i \cdot 2\sqrt{3}$ (b) 2 (with multiplicity 2) (c) $2 \pm i \cdot \sqrt{3}$ (d) $2 \pm i$
(e) A has no eigenvalues

$$\det \begin{bmatrix} 2-\lambda & 3 \\ -1 & 2-\lambda \end{bmatrix} = (2-\lambda)^2 - 3 \cdot (-1) = (2-\lambda)^2 + 3 = 0$$

$$\Updownarrow$$

$$2-\lambda = \pm\sqrt{3}i$$

$$\Updownarrow$$

$$\boxed{\lambda = 2 \pm \sqrt{3}i}$$

6. Consider the equation

$$y'' - 2ty' + e^t y = 0.$$

Find the Wronskian of the fundamental set of solutions of this equation determined by the conditions $y_1(0) = 2$, $y_1'(0) = 1$ and $y_2(0) = -1$, $y_2'(0) = 3$.

(a) e^{t+7} (b) $7e^{t^2}$ (c) $-t^2 + 7$ (d) 7 (e) $(t+7)e^t$

$$W(0) = \det \begin{bmatrix} y_1(0) & y_2(0) \\ y_1'(0) & y_2'(0) \end{bmatrix} = \det \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = 7$$

Abel's Theorem

$$W(t) = c \cdot e^{-\int p(t)dt}, \quad \boxed{p(t) = -2t}$$

$$-\int -2t\,dt = -(-t^2) = t^2$$

$$\left.\begin{aligned} W(t) &= c \cdot e^{t^2} \\ W(0) &= 7 \end{aligned}\right] \Rightarrow c = 7$$

$$\Rightarrow \boxed{W(t) = 7e^{t^2}}$$

7. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & -3 \\ 1 & -1 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Find the invertible matrix P such that $A = PDP^{-1}$.

(a) $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 2 \\ 1 & 1 & -3 \\ 2 & 2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 1 & 1 \\ -3 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -3 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

(e) P cannot be determined from the given data.

eigenvalues: $1, -1, 3$

eigenspaces: $E_1 = \text{Nul}(A - I) = \text{Nul}\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & -3 \\ 1 & -1 & -1 \end{bmatrix} = \text{Nul}\begin{bmatrix} 1 & 1 & -3 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad x_1 = 2t,\ x_2 = t,\ x_3 = t$

$= \text{Span}\left\{\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}\right\}$ ← (a)

$E_{-1} = \text{Nul}\begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & -3 \\ 1 & -1 & 1 \end{bmatrix} = \text{Span}\left\{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right\}$

$E_3 = \text{Nul}\begin{bmatrix} -2 & 0 & 0 \\ 1 & -1 & -3 \\ 1 & -1 & -3 \end{bmatrix} = \text{Span}\left\{\begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}\right\}$

$P = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ (a)

8. Let $y(t)$ be the unique solution of the initial value problem

$$\ln t \cdot \frac{dy}{dt} - \frac{2y}{\cos t} = \frac{t^2}{2^t - 8} \qquad y(2) = \pi$$

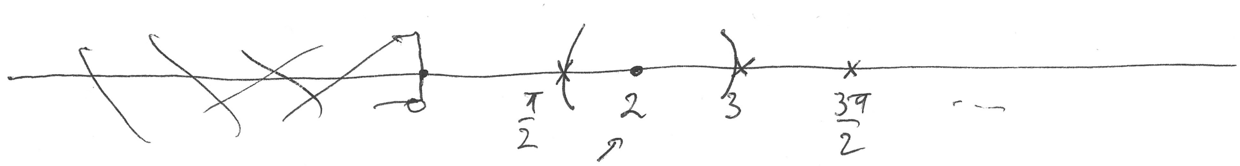
What is the largest interval on which a solution y is guaranteed to exist?

(a) $t > 0$ (b) $\frac{\pi}{2} < t < 3$ (c) $t < 1$ (d) $1 < t < 3\pi/2$ (e) $3 < t < \frac{3\pi}{2}$

$$\frac{dy}{dt} - \frac{2}{\ln t \cdot \cos t}\, y = \frac{t^2}{\ln t\,(2^t - 8)}$$

bad: $t \le 0$, $t = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

bad: $t \le 0$, $2^t = 8 \Leftrightarrow t = 3$



$I = (\frac{\pi}{2}, 3)$

9. Which of the following statements is **not** true for an invertible $n \times n$ matrix?

(a) $\dim(\text{Row } A) = n$

(b) $\text{rank } A = n$

(c) 0 is an eigenvalue of A

(d) $A^t A^{-1}$ is invertible

(e) $\dim(\text{Nul } A) = 0$

0 is an eigenvalue $\Rightarrow$ $\det A = 0$

NOT invertible

10. Which formula describes the general solution of the differential equation

$$2t^2 y'' + 3ty' - y = 0, \ t > 0$$

given the fact that $y_1(t) = t^{-1}$ is a solution of this equation.

(a) $c_1 t^2 + c_2 t^{-1}$ (b) $c_1 t^{-1} + c_2$ (c) $c_1 t^{-1} + c_2 t^{2/3}$ (d) $c_1 e^t + c_2 t^{-1}$ (e) $c_1 t^{-1} + c_2 t^{1/2}$

$$y'' + \underbrace{\frac{3}{2t}}_{p} y' - \frac{1}{2t^2} y = 0$$

Reduction of order: $y_2 = v \cdot y_1$ where $w = v'$ satisfies

$$y_1 \cdot w' + (2y_1' + p y_1) w = 0$$

$y_1 = t^{-1}$

$y_1' = -t^{-2}$

$\Leftrightarrow t^{-1} w' + \left(-2t^{-2} + \frac{3}{2} t^{-2}\right) w = 0$

$\Leftrightarrow t^{-1} w' - \frac{1}{2} t^{-2} w = 0$

$\Leftrightarrow \int \frac{dw}{w} = \int \frac{1}{2} t^{-1} dt \Leftrightarrow \ln w = \frac{1}{2} \ln t$

$\Leftrightarrow w = t^{1/2} \Rightarrow v = \frac{2t^{3/2}}{3} \Rightarrow y_2 = \frac{2}{3} t^{1/2}$

General solution

$c_1 t^{-1} + \underbrace{\left(c_2 \cdot \frac{2}{3}\right)}_{\text{constant}} t^{1/2}$

e

11. Find the general solution of

$$y'' + 2y' + \frac{13}{4}y = 0$$

(a) $y(t) = c_1e^{-t} + c_2(\cos(3t/2) + \sin(3t/2))$ (b) $y(t) = c_1te^{-t} + c_2e^{-t}$
(c) $y(t) = c_1e^{-t}\cos(3t/2) + c_2e^{-t}\sin(3t/2)$ (d) $y(t) = c_1e^{-t} + c_2e^{3t/2}$
(e) $y(t) = c_1\cos(-t) + c_2\sin(3t/2)$

$r^2 + 2r + \frac{13}{4} = 0$

$(r+1)^2 = -\frac{9}{4}$

$r = -1 \pm \frac{3}{2}i$

FSS $\{e^{-t}\cos(3t/2), e^{-t}\sin(3t/2)\} \Rightarrow$ (c)

12. Which formula describes implicitly the solution of the initial value problem

$$3e^x \cdot \frac{dy}{dx} - \frac{x}{y^2} = 0, \quad y(0) = 1.$$

(a) $3ye^x = x^2 + 3$ (b) $3e^x = \frac{x}{y} + 3$ (c) $e^x(x+y) = y^2$
(d) $y^3 + (x+1)e^{-x} = 2$ (e) $y^3 + 2y = 3e^x + x$

$$\int y^2\,dy = \int \frac{x}{3e^x}\,dx = \frac{1}{3}\int x e^{-x}\,dx$$

$$\int y^2\,dy = \frac{y^3}{3}$$

$$= \frac{1}{3}\left(-x \cdot e^{-x} - \int -e^{-x}\,dx\right)$$

$$= \frac{1}{3}\left(-xe^{-x} - e^{-x}\right) + C$$

$$\Rightarrow \frac{y^3 + (x+1)e^{-x}}{3} = C = \frac{2}{3} \Rightarrow \boxed{y^3 + (x+1)e^{-x} = 2}$$

$x = 0$, $y = 1$: $\frac{1+1}{3} = C$, $\boxed{C = \frac{2}{3}}$

13. Consider the matrices

$$A = \begin{bmatrix} 2 & 4 & -2 & 1 & 11 \\ 3 & 6 & -3 & 1 & 15 \\ -1 & -2 & 1 & 2 & 2 \\ 4 & 8 & -4 & 4 & 28 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where B is the reduced echelon form of A. A basis for the orthogonal complement of the row space of A is given by

(a) $\left\{ \begin{bmatrix} 2 \\ 3 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 4 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ -3 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ -4 \\ 4 \\ 28 \end{bmatrix} \right\}$ (e) $\left\{ \begin{bmatrix} 4 \\ 6 \\ -2 \\ 8 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 11 \\ 15 \\ 2 \\ 28 \end{bmatrix} \right\}$

$(\text{Row } A)^\perp = \text{Nul } A$

$x_1 = -2s + t - 4u$
$x_2 = s$
$x_3 = t$
$x_4 = -3u$
$x_5 = u$

$$\vec{x} = s\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + u\begin{bmatrix} -4 \\ 0 \\ 0 \\ -3 \\ 1 \end{bmatrix}$$

(b)

14. Based on the method of Undetermined Coefficients, find the form of a particular solution of the differential equation

$$y'' + 4y' + 5y = (t^2 + 1)e^{-2t}.$$

$r^2 + 4r + 5 = 0$
$(r+2)^2 = -1$
$r = -2 \pm i$

(a) $Y(t) = A_0(t^2+1)e^{-2t}\cos(t) + B_0(t^2+1)e^{-2t}\sin(t)$
(b) $Y(t) = (A_0t^2 + A_1t + A_2)e^{-2t}\cos(t) + (B_0t^2 + B_1t + B_2)e^{-2t}\sin(t)$
(c) $Y(t) = t(A_0t^2 + A_1t + A_2)e^{-2t}$
(d) $Y(t) = (A_0t^2 + A_1t + A_2)e^{2t} + (B_0t^2 + B_1t + B_2)e^{-2t}$
(e) $Y(t) = (A_0t^2 + A_1t + A_2)e^{-2t}$

FSS $\{e^{-2t}\cos t, e^{-2t}\sin t\}$

$t^s(A_0t^2 + A_1t + A_2)e^{-2t}$

$s =$ # times -2 is a root of char. eq $= 0$

15. The second column of the inverse of the matrix $\begin{bmatrix} 1 & -2 & 1 \\ 2 & -3 & 2 \\ -1 & 2 & 0 \end{bmatrix}$ is

(a) $\begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$ (b) $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ (e) $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 2 & -3 & 2 & 1 \\ -1 & 2 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

d

16. Consider the initial value problem

$$\frac{dy}{dt} = 2y^2 - 4y, \qquad y(5) = 1.$$

Which of the following describes the nature of the solution?

(a) $\lim_{t\to-\infty} y(t) = 2$; $\lim_{t\to\infty} y(t) = 0$; inflection point at $y = 1$

(b) $\lim_{t\to-\infty} y(t) = 2$; $\lim_{t\to\infty} y(t) = \infty$; concave up

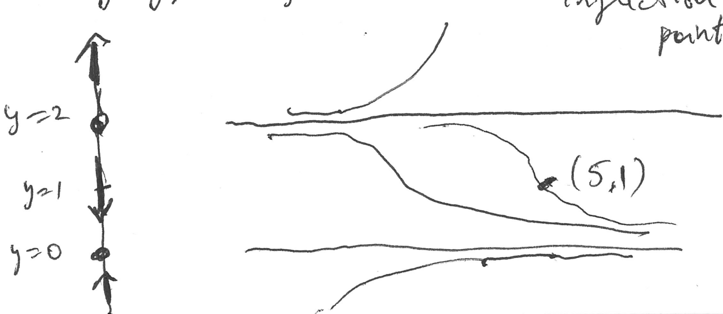
(c) $\lim_{t\to-\infty} y(t) = 0$; $\lim_{t\to\infty} y(t) = 4$; inflection point at $y = 2$

(d) $\lim_{t\to-\infty} y(t) = -\infty$; $\lim_{t\to\infty} y(t) = 0$; concave down

(e) $\lim_{t\to-\infty} y(t) = 0$; $\lim_{t\to\infty} y(t) = -\infty$; inflection point at $y = 1/2$

$f(y) = 2y^2 - 4y = 2y(y-2)$ critical points $y=0$, $y=2$

$f'(y) = 4y - 4$ inflection points $y=1$, and



17. Recall that $\mathbb{P}_n$ denotes the vector space of polynomials of degree at most n, and consider the linear transformation $T : \mathbb{P}_2 \to \mathbb{P}_3$ defined by

$$T(y) = ty'' - y' + (t+1)y.$$

The matrix of T relative to the basis $\{1, t, t^2\}$ of $\mathbb{P}_2$ and the basis $\{1, t, t^2, t^3\}$ of $\mathbb{P}_3$ is

(a) $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} t \\ -1 \\ t+1 \end{bmatrix}$ (c) $\begin{bmatrix} 1-t \\ 1+t \\ t+t^2 \\ t^2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(e) it cannot be determined from the given information.

$T(1) = t+1$

$T(t) = -1 + (t+1)t = t^2 + t - 1$

$T(t^2) = 2t - 2t + (t+1)t^2 = t^3 + t^2$

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

18. Solve the initial value problem

$$\begin{cases} ty' + (t+1)y = te^{-t}, \ t > 0 \\ y(1) = 2e^{-1} \end{cases}$$

(a) $2e^{-t}$ (b) $te^{-t} + 1$ (c) $(t^2+1)e^{-t}$ (d) $\dfrac{1+t}{e^t}$ (e) $\dfrac{t^2+3}{2te^t}$

$y' + \underbrace{\left(1 + \frac{1}{t}\right)}_{p(t)} y = \underbrace{e^{-t}}_{g(t)}$

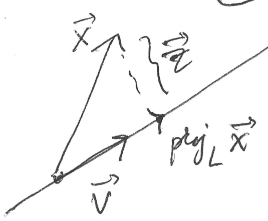
Integrating factor $\mu(t) = e^{\int p(t)\,dt} = e^{\int 1 + \frac{1}{t}\,dt} = e^{t + \ln t} = e^t \cdot t$

$y = \dfrac{\int \mu(t) \cdot g(t)\,dt}{\mu(t)} = \dfrac{\int e^t \cdot t \cdot e^{-t}\,dt}{e^t \cdot t} = \dfrac{t^2/2 + C}{e^t \cdot t} = \dfrac{t^2+3}{2te^t}$ (e)

$y(1) = \dfrac{1/2 + C}{e} = \dfrac{2}{e} \Rightarrow \boxed{C = \frac{3}{2}}$

19. Consider the lined L spanned by the vector $\vec{v} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$. The distance from the vector $\vec{x} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$ to the line L is

(a) $\sqrt{45}$ (b) $\sqrt{5}$ (c) $2\sqrt{3}$ (d) 5 (e) $\sqrt{50}$



$$\text{proj}_L \vec{x} = \frac{\vec{x}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\cdot\vec{v} = \frac{15}{5}\cdot\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$$

$$\vec{z} = \vec{x} - \text{proj}_L \vec{x} = \begin{bmatrix} 7 \\ -1 \end{bmatrix} - \begin{bmatrix} 6 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\text{dist}(\vec{x}, L) = \|\vec{z}\| = \sqrt{1^2+2^2} = \sqrt{5}$$

20. Using the method of Variation of Parameters, find a particular solution of the differential equation

$$x^2y'' - 3xy' + 4y = x^2\ln(x), \ x > 0,$$

knowing that $\{y_1, y_2\} = \{x^2, x^2\ln(x)\}$ is a fundamental set of solutions for the homogeneous equation $x^2y'' - 3xy' + 4y = 0$.

(a) $x\ln(x) + \frac{x^3}{3}$ (b) $\frac{x^3\ln(x)}{2}$ (c) $\frac{x^2\ln^3(x)}{6}$ (d) $2x\ln^2(x)$ (e) $\frac{(x+\ln(x))^2}{2}$

$Y = u_1y_1 + u_2y_2$

$$y'' \underbrace{-\frac{3}{x}}_{p(x)} y' + \underbrace{\frac{4}{x^2}}_{q(x)} y = \underbrace{\ln x}_{g(x)}$$

$$W = \det\begin{bmatrix} x^2 & x^2\ln x \\ 2x & 2x\ln x + \frac{x^2}{x} \end{bmatrix}$$

$$= \cancel{2x^3\ln x} + x^3 - \cancel{2x^3\ln x}$$

$$= x^3$$

$$u_1 = \int \frac{-y_2\cdot g}{W}\,dx = \int \frac{-x^2\cdot\ln^2 x}{x^3}\,dx = \int -u^2\,du = -\frac{u^3}{3}$$

$$u_2 = \int \frac{y_1\cdot g}{W}\,dx = \int \frac{x^2\cdot\ln x}{x^3}\,dx = \int u\,du = \frac{u^2}{2}$$

$$\boxed{\begin{aligned} u &= \ln x \\ du &= \frac{dx}{x} \end{aligned}}$$

$$u_1 = -\frac{\ln^3 x}{3} \qquad u_2 = \frac{\ln^2 x}{2}$$

Particular: $Y = -\frac{\ln^3 x}{3}\cdot x^2 + \frac{\ln^2 x}{2}\cdot x^2\ln x$

$$= \frac{x^2\cdot\ln^3 x}{6}$$

(c)