

M20580 L.A. and D.E. Tutorial
Quiz 1

1. Determine whether the vector \mathbf{w} can be written as a linear combination of the vectors \mathbf{v}_1 , \mathbf{v}_2 , and \mathbf{v}_3 . If yes, find scalars a_1 , a_2 , a_3 such that $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 = \mathbf{w}$.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 6 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 3 \\ 4 \\ -9 \end{bmatrix}, \text{ and } \mathbf{w} = \begin{bmatrix} 3 \\ -7 \\ -4 \end{bmatrix}$$

Solution: To solve $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 = \mathbf{w}$, row reduce the corresponding augmented matrix:

$$\begin{bmatrix} 1 & -2 & 3 & 3 \\ 0 & 1 & 4 & -7 \\ -3 & 6 & -9 & -4 \end{bmatrix} \xrightarrow{R_3+3R_1} \begin{bmatrix} 1 & -2 & 3 & 3 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

By the third row, $0a_1 + 0a_2 + 0a_3 = 5$, which is impossible. Thus, \mathbf{w} cannot be written as a linear combination of the vectors \mathbf{v}_1 , \mathbf{v}_2 , and \mathbf{v}_3 .

2. Determine the general solution to the system of equations below.

$$\begin{aligned} 8x_1 + 5x_2 &= 5 \\ 2x_1 + x_2 &= 3. \end{aligned}$$

Solution: To solve the system of equations, set up an augmented matrix, and row reduce.

$$\begin{bmatrix} 8 & 5 & 5 \\ 2 & 1 & 3 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 2 & 1 & 3 \\ 8 & 5 & 5 \end{bmatrix} \xrightarrow{R_2 - 4R_1} \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & -7 \end{bmatrix} \xrightarrow{R_1 - R_2} \begin{bmatrix} 2 & 0 & 10 \\ 0 & 1 & -7 \end{bmatrix}$$

From this, we see $x_2 = -7$ and $x_1 = 5$.