M20580 L.A. and D.E. Tutorial
Quiz 10

1. Find the general solution to the following differential equation:

$$y'' - 5y' + 6y = 0$$

Solution: This is a homogeneous second order differential equation with constant coefficients. Therefore, we first need to find the auxiliary equation:

$$m^2 - 5m + 6 = (m - 3)(m - 2) = 0$$

It immediately follows that the general solution is:

$$y = c_1 e^{3x} + c_2 e^{2x}$$

2. Find the general solution to the following differential equation:

$$2xy + (x^2 + 2y)y' = 0$$

Solution: By simply rewriting the equation as:

$$2xy\,dx + (x^2 + 2y)dy = 0$$

We see that this a differential equation of the form $M(x, y)dx + N(x, y)dy = 0$, so we first need to see if it is exact:

$$M_y = 2x = N_x$$

Since it is exact, we can simply integrate to obtain the solution:

$$f(x, y) = \int M(x, y)dx = \int 2xy\,dx = x^2 y + g(y)$$
$$\Rightarrow N(x, y) = x^2 + 2y = x^2 + g'(y) \Rightarrow g'(y) = 2y \Rightarrow g(y) = y^2$$

Therefore, our final answer is:

$$f(x, y) = x^2 y + y^2 = C$$