Name: SOLUTIONS Date: 02/02/2023

M20580 L.A. and D.E. Tutorial
Quiz 2

1. For each one of the following matrices, determine whether the matrix is invertible, and find its inverse if it exists.

(a) $\begin{bmatrix} 5 & -1 \\ 3 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & 10 \\ -1 & 5 \end{bmatrix}$

Solution: Recall that a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible if $ad - bc \neq 0$, and in this case the inverse is given by

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

If $ad - bc = 0$, A is not invertible.

(a) Since $5(4) - (-1)(3) = 23 \neq 0$, this matrix is invertible and its inverse is given by

$$A^{-1} = \frac{1}{23} \begin{bmatrix} 4 & 1 \\ -3 & 5 \end{bmatrix}.$$

(b) Since $-2(5) - 10(-1) = 0$, this matrix is not invertible.

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} y^2 \\ x \end{bmatrix}.$$

Determine whether T is a linear transformation.

Solution: The transformation T is a linear transformation if $T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u})+T(\mathbf{v})$ and $T(c\mathbf{u}) = cT(\mathbf{u})$ for $\mathbf{u}, \mathbf{v} \in \mathbb{R}^2$ and $c \in \mathbb{R}$. Notice that

$$T\left(c\begin{bmatrix} x \\ y \end{bmatrix}\right) = T\left(\begin{bmatrix} cx \\ cy \end{bmatrix}\right) = \begin{bmatrix} c^2y^2 \\ cx \end{bmatrix} = c\begin{bmatrix} cy^2 \\ x \end{bmatrix} \neq c\begin{bmatrix} y^2 \\ x \end{bmatrix} = cT\left(\begin{bmatrix} x \\ y \end{bmatrix}\right).$$

Hence, T cannot be a linear transformation. You can also check that the property $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ is not satisfied by T.