Name: Date: 02/09/2023

M20580 L.A. and D.E. Tutorial

Quiz 3

1. Determine if the following vectors are linearly independent:

$$\begin{bmatrix}1\\2\\1\\3\end{bmatrix}, \quad \begin{bmatrix}1\\3\\1\\2\end{bmatrix}, \quad \begin{bmatrix}-2\\-1\\0\\1\end{bmatrix}$$

> ***Solution:*** The matrix with rows formed by the vectors above can be reduced to the following matrix:
>
> $$\begin{bmatrix}1 & 2 & 1 & 3\\ 1 & 3 & 1 & 2\\ -2 & -1 & 0 & 1\end{bmatrix} \sim \begin{bmatrix}1 & 2 & 1 & 3\\ 0 & 1 & 0 & -1\\ 0 & 3 & 2 & 7\end{bmatrix} \sim \begin{bmatrix}1 & 2 & 1 & 3\\ 0 & 1 & 0 & -1\\ 0 & 0 & 2 & 10\end{bmatrix}$$
>
> Hence we could see that the rank of the matrix is 3. Therefore, the given above three vectors are linearly independent.

2. Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear transformation given by:

$$T\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}x_1 - x_2\\x_1 + x_2\end{bmatrix}, \text{ for all } \begin{bmatrix}x_1\\x_2\end{bmatrix} \in \mathbb{R}^2$$

Find the standard matrix for the linear transformation T, i.e., find a 2×2 matrix A such that $T(\mathbf{x}) = A\mathbf{x}$.

> ***Solution:*** (a) Since $T\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}, \quad T\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-1\\1\end{bmatrix}$, so $A = \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$.