Name: Date: 02/23/2023

M20580 L.A. and D.E. Tutorial
Quiz 4

1. Find the change of basis matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$ where two bases are

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\} \quad \text{and} \quad \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}.$$

Solution: We get the augmented matrix which can be reduced as follows

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 2 \\ 1 & 2 & -1 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 1 & 1 & 2 \\ 0 & 1 & -2 & -1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & 3 & 3 \\ 0 & 1 & -2 & -1 \end{array}\right].$$

Thus,

$$P_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{bmatrix} 3 & 3 \\ -2 & -1 \end{bmatrix}.$$

2. Let $p(x) = 1 - 2x, q(x) = x + 2x^2$, $r(x) = -2 + 3x - 2x^2$, and $s(x) = 1 + x + 6x^2$. Find real numbers a, b, and c such that $s(x) = ap(x) + bq(x) + cr(x)$ if they exist.

Solution: The equation $ap(x) + bq(x) + cr(x) = s(x)$ give a linear system whose augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ -2 & 1 & 3 & 1 \\ 0 & 2 & -2 & 6 \end{array}\right]$$

This can be reduced to

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

which means that there are infinitely many solutions. We can choose $c = 0, b = 3$, and $a = 1$ so that $ap(x) + bq(x) + cr(x) = s(x)$.