Name: Date: 03/23/2023

M20580 L.A. and D.E. Tutorial
Quiz 6

1. Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 3 & 0 & 5 \\ 0 & 1 & -6 \\ 0 & 0 & 8 \end{bmatrix}.$$

Is A diagonalizable? Justify your answer.

> ***Solution:*** The eigenvalues of an upper triangular matrix are the entries along its diagonal. In other words,
>
> $$\det(A - \lambda I) = \det \begin{bmatrix} 3-\lambda & 0 & 5 \\ 0 & 1-\lambda & -6 \\ 0 & 0 & 8-\lambda \end{bmatrix} = (3-\lambda)(1-\lambda)(8-\lambda).$$
>
> Hence, the eigenvalues of A are $\lambda = 3, 1, 8$. Since A has three distinct eigenvalues, A is diagonalizable.

2. The eigenvalues of $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ are $\lambda_1 = 1$ and $\lambda_2 = 3$. Find a basis for the eigenspace for each eigenvalue.

> ***Solution:*** The eigenspace of an eigenvalue λ is the nullspace of
>
> $$A - \lambda I = \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}.$$
>
> Plugging in for λ, we obtain
>
> $$\text{Nullspace}(A - I) = \text{Span}\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\} = \text{Eigenspace}(\lambda_1)$$
>
> $$\text{Nullspace}(A - 3I) = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\} = \text{Eigenspace}(\lambda_2)$$