Name: Date: 03/30/2023

M20580 L.A. and D.E. Tutorial
Quiz 7

1. Determine whether the following set of vectors is an orthogonal basis for $\mathbb{R}^3$.

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \right\}.$$

Solution: Let $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$. Then

$$\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= 1(1) + 1(-1) + 1(0) = 0, \\ \mathbf{u} \cdot \mathbf{w} &= 1(1) + 1(1) + 1(-2) = 0, \\ \mathbf{v} \cdot \mathbf{w} &= 1(1) + (-1)(1) + 0(-2) = 0. \end{aligned}$$

Hence, the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ form an orthogonal set. By Theorem 5.1 in Poole's book, they are linearly independent. In consequence, they span $\mathbb{R}^3$ and form an orthogonal basis for $\mathbb{R}^3$.

2. Let $\mathbf{u} = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} \in \mathbb{R}^3$.

(a) Find $\mathbf{u} \cdot \mathbf{v}$.

Solution: $\mathbf{u} \cdot \mathbf{v} = 2(0) + 0(3) + (-3)(-1) = 3$.

(b) Find a unit vector in the direction of $\mathbf{u}$.

Solution: Recall that $\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}} = \sqrt{2(2) + 0(0) + (-3)(-3)} = \sqrt{13}$. Hence, a unit vector in the direction of $\mathbf{u}$ is $\mathbf{w} = \dfrac{1}{\sqrt{13}}\mathbf{u}$.

(c) Find $\text{proj}_{\mathbf{u}}\mathbf{v}$.

Solution: $\text{proj}_{\mathbf{u}}\mathbf{v} = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u} = \dfrac{3}{13}\mathbf{u}$.