

**M20580 L.A. and D.E. Tutorial**  
Quiz 9

1. Find  $QR$  factorization of a matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 4 \\ 4 & 0 & -3 \end{bmatrix}.$$

**Solution:** Since the column of  $A$  is already orthogonal,

$$Q = \begin{bmatrix} 0 & 1 & 0 \\ 3/5 & 0 & 4/5 \\ 4/5 & 0 & -3/5 \end{bmatrix}$$

and

$$\begin{aligned} R &= Q^T A \\ &= \begin{bmatrix} 0 & 3/5 & 4/5 \\ 1 & 0 & 0 \\ 0 & 4/5 & -3/5 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 4 \\ 4 & 0 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}. \end{aligned}$$

2. Given the following differential equation

$$xyy' = x.$$

- What is the order of this differential equation?
- Is it linear?
- What is a solution of this differential equation?

**Solution:** This is a nonlinear differential equation of order 1. To solve for a solution of this differential equation, we can use separation of variable method. That is we have

$$yy' = 1.$$

Then integrate both sides with respect to  $x$ , we get

$$\frac{1}{2}y^2 = x + C.$$

Then  $y^2 = 2x + 2C$ .