

Review Session

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1. Determine the values of a for which the system $Ax = b$ is consistent, where

$$A = \begin{bmatrix} a & 1 & 1 & 0 \\ 0 & a & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

and

$$\text{a) } b = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}. \qquad \text{b) } b = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}.$$

2. Consider the linear map $T : \mathbb{P}_2 \rightarrow \mathbb{R}^4$ given by

$$T(p) := \begin{bmatrix} p(0) + p'(0) \\ p(0) - p'(0) \\ p(1) \\ p'(1) \end{bmatrix}.$$

- (a) Write the matrix of T in the standard bases.
(b) Find bases for the kernel and image of T .
3. Determine if the matrix

$$\begin{bmatrix} 4 & -1 & -2 \\ 2 & 0 & -1 \\ 4 & -2 & -2 \end{bmatrix}$$

is diagonalizable. If so, diagonalize it.

4. Solve the differential equation

$$y'' + 3y' + 2y = \sin(e^x) - e^{-x}.$$

5. Solve the differential equation

$$y^{(4)} + 2y^{(2)} + y = x \cos(x).$$