

Worksheet 1

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August 30, 2010

Solve the systems of equations

$$1. \begin{cases} x_1 - 5x_2 + 4x_3 = -3 \\ 2x_1 - 7x_2 + 3x_3 = -2 \\ -2x_1 + x_2 + 7x_3 = -1 \end{cases} \quad 2. \begin{cases} 2x_1 - 4x_3 = -10 \\ x_2 + 3x_3 = 2 \\ 3x_1 + 5x_2 + 8x_3 = -6 \end{cases}$$

Determine if the following systems are consistent:

$$3. \begin{cases} x_1 - 6x_2 = 5 \\ x_2 - 4x_3 + x_4 = 0 \\ -x_1 + 6x_2 + x_3 + 5x_4 = 3 \\ -x_2 + 5x_3 + 4x_4 = 0 \end{cases} \quad 4. \begin{cases} x_1 - 2x_3 = -1 \\ x_2 - x_4 = 2 \\ -3x_2 + 2x_3 = 0 \\ -4x_1 + 7x_4 = -5 \end{cases}$$

Find the general solutions of the systems whose augmented matrices are given below:

$$5. \begin{bmatrix} 1 & 0 & 2 & 5 \\ 2 & 0 & 3 & 6 \end{bmatrix} \quad 9. \begin{bmatrix} 1 & 0 & 0 & 8 & -3 \\ 0 & 1 & 0 & 4 & -6 \\ 0 & 0 & 1 & -7 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$6. \begin{bmatrix} 1 & 3 & 6 & 9 \\ -1 & 1 & -2 & -1 \end{bmatrix} \quad 10. \begin{bmatrix} 1 & 0 & -9 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
$$7. \begin{bmatrix} 1 & 2 & 4 \\ -2 & -3 & -5 \\ 2 & 1 & -1 \end{bmatrix}$$
$$8. \begin{bmatrix} 2 & -4 & 3 \\ -6 & 12 & -9 \\ 4 & -8 & 6 \end{bmatrix}$$

Determine the value(s) of h such that the given matrix is the augmented matrix of a consistent linear system.

$$11. \begin{bmatrix} 1 & 4 & 2 \\ -3 & h & -1 \end{bmatrix} \quad 12. \begin{bmatrix} 1 & -3 & 1 \\ h & 6 & -2 \end{bmatrix}$$

13. Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

14. Find a condition on g, h, k that makes the following system consistent.

$$\begin{cases} x_1 - 4x_2 + 7x_3 = g \\ 3x_2 - 5x_3 = h \\ -2x_1 + 5x_2 - 9x_3 = k \end{cases}$$

15. Show that the system

$$\begin{cases} a_0 + a_1 \cdot 1 + a_2 \cdot 1^2 = g \\ a_0 + a_1 \cdot 2 + a_2 \cdot 2^2 = h \\ a_0 + a_1 \cdot 3 + a_2 \cdot 3^2 = k \end{cases}$$

is consistent and has a unique solution for all values of g, h, k . What happens if you replace 1, 2, 3 by 5, 72 and 101?