Worksheet 12

Claudiu Raicu

November 12, 2010

1. Find a general solution of the system $x' = Ax$ for $A = \begin{bmatrix} -2 & -5 \\ 1 & 2 \end{bmatrix}$.

2. Find a fundamental matrix for the system $x' = Ax$ for $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$.

(3-4) Use undetermined coefficients to find a general solution to the system $x'(t) = Ax(t) + f(t)$, where

3. $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$, $f(t) = \begin{bmatrix} -t-1 \\ -4t-2 \end{bmatrix}$.

4. $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, $f(t) = \begin{bmatrix} -4\cos(t) \\ -\sin(t) \end{bmatrix}$.

(5-6) Use variation of parameters to find a general solution of the system $x'(t) = Ax(t) + f(t)$, where

5. $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$, $f(t) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

6. $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $f(t) = \begin{bmatrix} 8\sin(t) \\ 0 \end{bmatrix}$.

7. Find the solution of the system

$$x'(t) = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \cdot x(t) + \begin{bmatrix} e^t \\ -e^t \end{bmatrix},$$

where $x(-1) = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$.

(8-10) Determine the matrix exponential e^{At}.

8. $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix}$.

10. $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$.

9. $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$.