Worksheet 13

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1. Determine the solutions, if any, to the boundary value problem

   a) $y'' + y = 0$, $0 < x < 2\pi$, $y(0) = 0$, $y(2\pi) = 1$.

   b) $y'' + y = 0$, $0 < x < 2\pi$, $y(0) = 1$, $y(2\pi) = 1$.

2. Determine the values of λ for which the given problem has a nontrivial solution. Also determine the corresponding nontrivial solutions.

   a) $y'' + \lambda y = 0$, $0 < x < \pi$, $y(0) = 0$, $y'(\pi) = 0$.

   b) $y'' + \lambda y = 0$, $0 < x < \pi$, $y(0) - y'(0) = 0$, $y(\pi) = 0$.

3. Solve the heat-flow problem

$$\begin{cases} \dfrac{\partial u}{\partial t}(x,t) = \beta \dfrac{\partial^2 u}{\partial x^2}(x,t), \ 0 < x < L, \ t > 0 \\ u(0,t) = u(L,t) = 0, \ t > 0 \\ u(x,0) = f(x), \ 0 < x < L. \end{cases}$$

   with $\beta = 3$, $L = \pi$ and

   a) $f(x) = \sin(x) - 6\sin(4x)$.

   b) $f(x) = \sin(x) - 7\sin(3x) + \sin(5x)$.

4. Solve the vibrating string problem

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2}(x,t) = \alpha^2 \dfrac{\partial^2 u}{\partial x^2}(x,t), \ 0 < x < L, \ t > 0 \\ u(0,t) = u(L,t) = 0, \ t \geq 0 \\ u(x,0) = f(x), \ 0 \leq x \leq L, \\ \dfrac{\partial u}{\partial t}(x,0) = g(x), \ 0 \leq x \leq L. \end{cases}$$

   with $\alpha = 3$, $L = \pi$ and

   a) $f(x) = 3\sin(2x) + 12\sin(13x)$, $g(x) \equiv 0$.

   b) $f(x) = 6\sin(2x) + 2\sin(6x)$, $g(x) = 11\sin(9x) - 14\sin(15x)$.

5. Compute the Fourier series

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos\left(\frac{n\pi x}{T}\right) + b_n \sin\left(\frac{n\pi x}{T}\right) \right\},$$

   for the given interval and function $f(x)$.

   a) $f(x) = |x|$, $-\pi < x < \pi$.

   b) $f(x) = \begin{cases} 1, & -2 < x < 0 \\ x, & 0 < x < 2 \end{cases}$.