Worksheet 14

Claudiu Raicu

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1. Compute the Fourier series for the function

   a) $f(x) = x$, $-\pi < x < \pi$.

   b) $f(x) = \begin{cases} -1, & -\pi < x < 0, \\ 1, & 0 < x < \pi. \end{cases}$

2. Find a formal solution to the given initial-boundary value problem

$$\begin{cases} \dfrac{\partial u}{\partial t}(x,t) = \beta \dfrac{\partial^2 u}{\partial x^2}(x,t) + P(x),\ 0 < x < L,\ t > 0 \\ u(0,t) = U_1,\ u(L,t) = U_2,\ t > 0 \\ u(x,0) = f(x),\ 0 < x < L. \end{cases}$$

   where

   a) $P(x) = 0$, $U_1 = 0$, $U_2 = 3\pi$, $\beta = 1$, $L = \pi$, $f(x) = 0$.

   b) $P(x) = e^{-x}$, $U_1 = 0$, $U_2 = 0$, $\beta = 1$, $L = \pi$, $f(x) = \sin(2x)$.

   c) $P(x) = 5$, $U_1 = 1$, $U_2 = 1$, $\beta = 3$, $L = \pi$, $f(x) = 1$.

3. Find a formal solution to the given initial-boundary value problem

$$\begin{cases} \dfrac{\partial u}{\partial t}(x,y,t) = \dfrac{\partial^2 u}{\partial x^2}(x,y,t) + \dfrac{\partial^2 u}{\partial y^2}(x,y,t),\ 0 < x,y < \pi,\ t > 0 \\ \dfrac{\partial u}{\partial x}(0,y,t) = \dfrac{\partial u}{\partial x}(\pi,y,t) = 0,\ 0 < y < \pi,\ t > 0, \\ u(x,0,t) = u(x,\pi,t) = 0, \\ u(x,y,0) = f(x,y),\ 0 < x,y < \pi. \end{cases}$$

   where

   a) $f(x,y) = \cos(x)\sin(y) + 4\cos(2x)\sin(y) - 3\cos(3x)\sin(4y)$.

   b) $f(x,y) = y$.

4. Find a formal solution to the vibrating string problem governed by the given initial-boundary value problem

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = \alpha^2 \dfrac{\partial^2 u}{\partial x^2}, & 0 < x < L,\ t > 0 \\ u(0,t) = u(L,t) = 0, & t > 0 \\ u(x,0) = f(x), & 0 < x < L \\ \dfrac{\partial u}{\partial t}(x,0) = g(x), & 0 < x < L \end{cases}$$

   where

   a) $\alpha = 2$, $L = \pi$, $f(x) = x^2(\pi - x)$, $g(x) = 0$.

   b) $\alpha = 3$, $L = \pi$, $f(x) = \sin(4x) + 7\sin(5x)$, $g(x) = \begin{cases} x, & 0 < x < \pi/2, \\ \pi - x, & \pi/2 < x < \pi \end{cases}$.

5. Find a solution to the initial value problem

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = \alpha^2 \dfrac{\partial^2 u}{\partial x^2}, & -\infty < x < \infty,\ t > 0 \\ u(x,0) = f(x), & -\infty < x < \infty, \\ \dfrac{\partial u}{\partial t}(x,0) = g(x), & -\infty < x < \infty. \end{cases}$$

where

a) $f(x) = x^2$, $g(x) = 0$.

b) $f(x) = g(x) = x$.

c) $f(x) = e^{-x^2}$, $g(x) = \sin(x)$.

6. Find a formal solution to the given boundary value problem

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0, & 0 < x < \pi,\ 0 < y < 1 \\ \dfrac{\partial u}{\partial x}(0,y) = \dfrac{\partial u}{\partial x}(\pi,y) = 0, & 0 \le y \le 1, \\ u(x,0) = \cos(x) - \cos(3x), & 0 \le x \le \pi, \\ u(x,1) = \cos(2x),\ 0 \le x \le \pi. \end{cases}$$