Worksheet 2

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Determine if the vectors are linearly independent

1. $\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix}.$

2. $\begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \end{bmatrix}.$

3. $\begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 5 \\ 4 \end{bmatrix}.$

4. $\begin{bmatrix} 4 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 8 \\ 1 \end{bmatrix}.$

5. How many pivot colummns must a 5×7 matrix have if its columns span $\mathbb{R}^5$?

6. How many pivot columns must a 7×5 matrix have if its columns are linearly independent?

7. Construct 3×2 matrices A and B such that $Ax = 0$ has only the trivial solution and $Bx = 0$ has a nontrivial solution.

8. Show that if v_1, v_2 are in $\mathbb{R}^4$ and v_2 is not a scalar multiple of v_1, then $\{v_1, v_2\}$ is linearly independent.

9. Show that if $v_1, \cdots, v_4$ are linearly independent vectors in $\mathbb{R}^4$, then $\{v_1, v_2, v_3\}$ is also linearly independent.

10. Show that if v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^4$ and v_4 is not a linear combination of v_1, v_2, v_3, then $\{v_1, v_2, v_3, v_4\}$ is linearly independent.

11. Show that if v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^3$ and v_4 is any vector in $\mathbb{R}^3$, then v_4 is a linear combination of v_1, v_2, v_3.

12. Suppose an $m \times n$ matrix A has n pivot columns. Explain why for each $\mathbf{b}$ in $\mathbb{R}^m$ the equation $Ax = \mathbf{b}$ has at most one solution.

13. Let
$$A = \begin{bmatrix} .5 & 0 & 0 \\ 0 & .5 & 0 \\ 0 & 0 & .5 \end{bmatrix}, \quad u = \begin{bmatrix} 1 \\ 0 \\ -4 \end{bmatrix}, \quad v = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$
Define $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T(x) = Ax$. Find $T(u)$ and $T(v)$.

With T defined by $T(x) = Ax$, find an x whose image under T is b, and determine if x is unique.

14. $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & -5 \\ -4 & 2 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ -5 \\ -6 \end{bmatrix}$

15. $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ 0 & 3 & -8 \end{bmatrix}$, $b = \begin{bmatrix} 5 \\ 9 \\ 4 \end{bmatrix}$

17. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation and let $\{v_1, v_2, v_3\}$ be a linearly dependent set in $\mathbb{R}^n$. Explain why the set $\{T(v_1), T(v_2), T(v_3)\}$ is linearly dependent.

18. Determine if the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^4$ given by $T(x_1, x_2, x_3, x_4) = (x_1 + x_2, x_2 + x_3, x_3 + x_4, 0)$ is onto. Is it one-to-one?