Worksheet 3

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Find the determinant and inverse (if it exists) of the matrix

1. $\begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$

3. $\begin{bmatrix} 5 & 10 \\ 4 & 7 \end{bmatrix}$

4. $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$

Find the determinants and inverses (if they exist) of the matrices

5. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix}$ Here the last matrix is a square matrix with n rows and n columns, and ones on and below the diagonal.

6. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & \dots & n \end{bmatrix}$ Here the last matrix is a square matrix with n rows and n columns, and with entries on and below the diagonal equal to i on the i-th column.

7. Let $A = \begin{bmatrix} -2 & -7 & -9 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{bmatrix}$. Find the third column of A^{-1} without computing the other columns.

8. Let $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$. Construct a 4×2 matrix D using only 1 and 0 as entries, such that $AD = I_2$. Is it possible that $CA = I_4$ for some 4×2 matrix C? Why or why not?

9. Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T(x_1, x_2) = (-5x_1 + 9x_2, 4x_1 - 7x_2).$$

Show that T is invertible and find a formula for T^{-1}.

10. Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T(x_1, x_2) = (6x_1 - 8x_2, -5x_1 + 7x_2).$$

Show that T is invertible and find a formula for T^{-1}.

11. Suppose that $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation with the property that $T(u) = T(v)$ for some pair of distinct vectors u and v. Can T map $\mathbb{R}^n$ onto $\mathbb{R}^n$? Why or why not?