

# Worksheet 9

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1. Verify that for  $b = 0$  and  $F_{ext}(t) = 0$ , the equation

$$my'' + by' + ky = F_{ext}(t)$$

has a solution of the form

$$y(t) = \sin(\omega t), \text{ where } \omega = \sqrt{k/m}.$$

2. An external force  $F(t) = 2 \cos(2t)$  is applied to a mass-spring system with  $m = 1$ ,  $b = 0$ , and  $k = 4$ , which is initially at rest; i.e.  $y(0) = y'(0) = 0$ . Verify that  $y(t) = \frac{1}{2}t \sin(2t)$  gives the motion of this spring. What will eventually (as  $t$  increases) happen to the spring?
3. Find a synchronous solution of the form  $A \cos(\Omega t) + B \sin(\Omega t)$  to the forced oscillator equation

$$y'' + 2y' + 4y = 3 \sin(5t), \text{ where } \Omega = 5.$$

Solve the differential equation

4.  $y'' - 4y' + y = 0$ .
5.  $y'' + 3y' = 0$ .

Solve the initial-value problem

6.  $2y'' + 5y' - 3y = 0$ ,  $y(0) = 1$ ,  $y'(0) = 4$ .
7.  $y'' - 2y' + 5y = 0$ ,  $y(\pi) = 0$ ,  $y'(\pi) = 2$ .

8. Determine whether the functions  $y_1 = te^{2t}$  and  $y_2(t) = e^{2t}$  are linearly dependent on the interval  $(0, 1)$ .

9. Find a general solution to the equations

(a)  $y''' - y'' + y' + 3y = 0$ .

(b)  $y''' + 2y'' + 5y' - 26y = 0$ .

10. Solve the initial-value problem

$$y''' - y' = 0, \quad y(0) = 2, \quad y'(0) = 3, \quad y''(0) = -1.$$