Points in multiprojective spaces and Hilbert functions of multigraded algebras

Favacchio Giuseppe
University of Catania

Abstract

The interpolation problem is a significant motivating problem in algebraic geometry and commutative algebra. It asks to describe all the polynomials that vanish at a collection of points in some ambient space, eventually with some restrictions.

The works of Geramita et al. give an answer for set of reduced points in a projective space. In the multigraded version of the interpolation problem the ambient space is a multiprojective space. Here the problem is still open, we have partial answers due to Giuffrida et al, Elena Guardo and Adam Van Tuyl for ACM sets of reduced or fat points in $\mathbb{P}^1 \times \mathbb{P}^1$.

A main obstacle is that the coordinate ring of a collection of points in a multiprojective space is not always Cohen-Macaulay. But, in most of all cases, the restriction to the CM case is also an interesting and still open problem. Moreover, a great limitation is the lack of a characterization theorem for the Hilbert function of multigraded algebras.

In this talk we survey on the multigraded version of the interpolation problem, we give a characterization of the Hilbert function of bigraded algebras in $k[\mathbb{P}^1 \times \mathbb{P}^1]$ and a description of ACM sets of reduced points in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$.