Title: Lefschetz properties and a problem on fat points

Abstract: Given general points  $p_1, \ldots, p_d$  in the projective plane and positive integers  $m_1, \ldots, m_d$ , "how many" curves of fixed degree j pass through the points with multiplicity at least  $m_i$  at  $p_i$ ? There is a natural guess, and the Segre-Harbourne-Gimigliano-Hirschowitz (SHGH) conjecture says what should be the only counterexamples to the natural guess. This can be interpreted as a conjecture giving the number of conditions on the complete linear system of plane curves of degree j imposed by the "fat point scheme"  $m_1p_1 + \cdots + m_dp_d$ . We extend this problem by replacing the complete linear system with the linear subsystem defined by a fixed set of points Z in the plane, and study the first interesting case. Our study involves line arrangements in the plane, and so-called "Lefschetz properties. This is joint work with David Cook II, Brian Harbourne and Uwe Nagel.