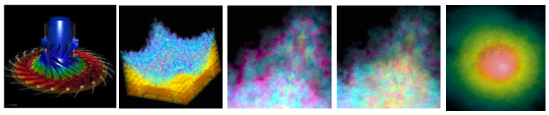
Information and Visualization

Han-Wei Shen

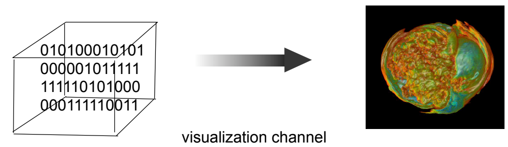
Department of Computer Science and Engineering

The Ohio State University



The Goal of Visualization

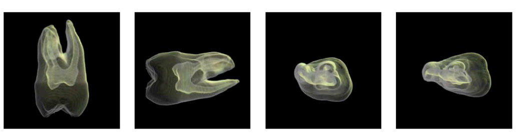
- The goal of visualization is to faithfully convey the maximal amount of information from the data through the display channel



How do we measure the information content?

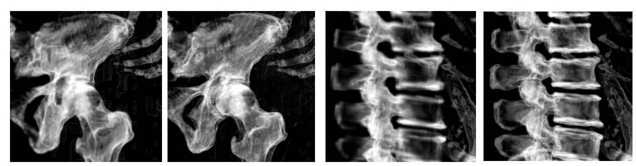
View Point Selection

- Due to 3D occlusion, images generated from different view will convey different amount of information
- How to choose views that can convey maximal amounts of information?



Multiresolution Visualization

- How do we measure and compare the quality of different LOD selections?
- Are the computation resources effectively utilized?



Low resolution High resolution Low resolution High resolution

Information Theory

- Study the fundamental limits to reliably transmit messages through a noisy channel
- Model the message as a random variable whose value is taken from a sequence of symbols
- Information content of the message is measured by Shannon's Information Entropy

Shannon Entropy

- The random variable takes a sequence of symbol $\{a_1, a_2, a_3, \ldots, a_n\}$ with probabilities $\{p_1, p_2, p_3, \ldots, p_n\}$
- The information contained in each symbol a_i is defined as:

$$\log(1/p_i) = -\log p_i$$

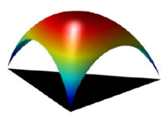
- The average amount of information expressed by the random variable is the entropy:

$$H(x) = -\sum_{i=1}^{n} p_i \log p_i$$

Properties of Entropy

- Entropy is to measure the average uncertainty of the random variable
- Entropy is a concave function, which has a maximum value when all outcomes are equally possible:

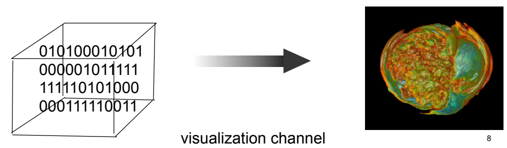
$$p_1 = p_2 = p_3 = ... = p_n$$



An example of three dimensional Probability vector {p1,p2,p3}

Information Theory and Visualization

- A data set can be considered as a random variable
- Each data point can be considered as an outcome for a random variable X
- We can measure the information content of the visualization output (image)

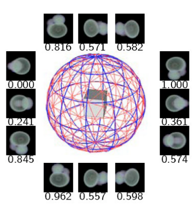


Finding a Good View

- Viewpoint entropy for a camera view:

$$H(x) = -\sum_{i=1}^{n} p_i \log p_i$$

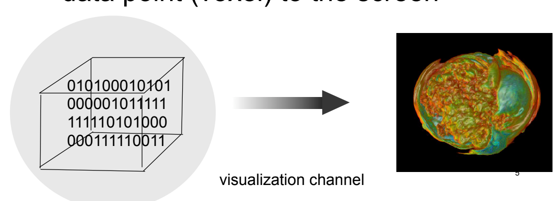
- Sample the view sphere and evaluate from multiple sample views



(Image courtesy: Takahashi et al)

Object Space Approach

- Consider that the visualization is generated from 3D data
- Evaluate the information content from each data point (voxel) to the screen



Information Content of a Voxel

- Each voxel's information content depends on its visibility and importance
  - visibility v: transparency from the camera to the voxel
  - importance w: the voxel's alpha value (defined by the transfer function)
- Visual probability for a voxel i:

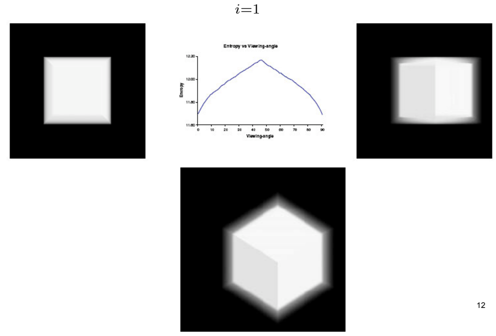
$$p_i = \frac{1}{\sigma} . \frac{v_i}{w_i} \quad , \text{ where } \quad \sigma = \sum_{i=0}^{n-1} \frac{v_i}{w_i}$$

- Information Content for the voxel:

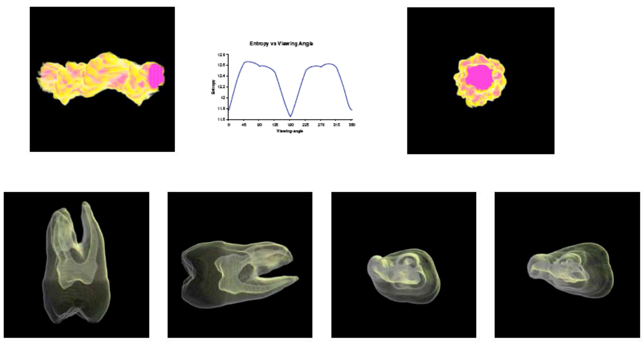
$$\log(1/p_i) = -\log p_i$$

Example

$$H(x) = -\sum_{i=1}^{n} p_i \log p_i$$



Examples



View Partitioning

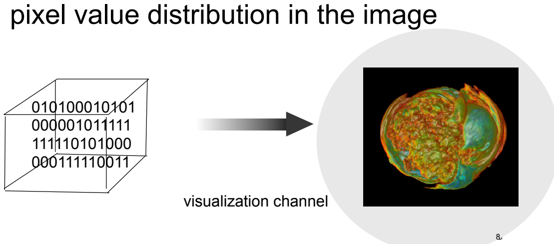
- Choose only one from the nearby similar view samples
- Use Jensen-Shannon (JS) divergence measure to estimate the distance between two view entropies q1 and q2

$$JS(q1, q2) = 2H(\frac{1}{2}q1 + \frac{1}{2}q2) - H(q1) - H(q2)$$

- Cluster the view samples based on the JS divergence

Image Space Approach

- Consider only the visualization output, i.e., the images
- Evaluate the information content based on the pixel value distribution in the image



View Selection Criteria

- Larger projection size
  - more voxels to be visible
- Even opacity distribution
  - opaque voxels do not clump together
- Even color distribution
  - data in different ranges can be seen
- More salient geometric features (curvature)

Projection & Opacity Distribution

- For an imagine contains n pixels with accumulated opacities $\{\alpha_0, \alpha_1, \alpha_2, ..., \alpha_{n-1}\}$, the probability of pixel i is defined as:

$$p_i = \frac{\alpha_i}{\sum_{i=0}^{n-1} \alpha_j}$$

- Background pixels are excluded
- The view entropy is defined in the same way:

$$H(x) = -\sum_{i=1}^{n} p_i \log p_i$$

Color Distribution

- A well designed transfer function should highlight salient features with attentive colors
- A good view should maximize the area of the salient colors while maintaining an even distribution
- Assuming there are n colors in the transfer function $\{C_0, C_1, ..., C_{n-1}\}$, with C_0 being the background
- For each pixel, we measure the perceptual color distance (using CIELUV) to the salient colors and classify the pixel

Entropy Calculation

- Calculate the area A_i for each color C_i
- The probability for C_i is defined as:

$$p_i = \frac{A_i}{T} \quad \text{where} \quad T = \sum_{i=0}^{n-1}(A_i)$$

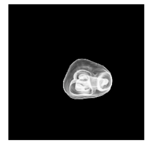
- The entropy for the image is calculated similarly:

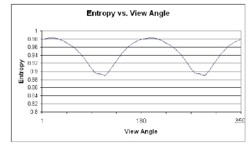
$$H(x) = -\sum_{i=1}^{n} p_i \log p_i$$

- The entropy of an image is a maximum when all salient colors are shown

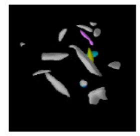
Examples

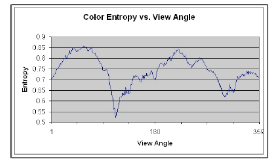
Opacity and Projection Area





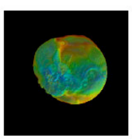
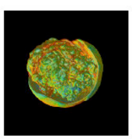
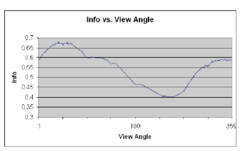
Color





Curvature Distribution

- Low curvatures imply flat areas and high curvatures mean highly irregular shapes.
- We can represent curvatures in a volume by color coding each voxel based on its curvature
- High luminance colors for high curvatures

The Final Utility Function

- Decide the best view based on opacity, color, and curvature

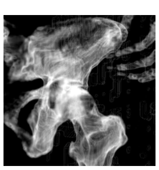
$$u(v) = \alpha.opacity(v) + \beta.color(v) + \gamma.curvature(v)$$

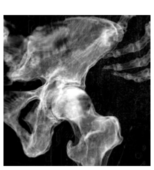
$$\alpha + \beta + \gamma = 1$$

- Normalize each component to [0,1]
- User can decide a different combination of weights and/or introduce new factors

Multiresolution Visualization

- How do we measure and compare the quality of different LOD selections?
- Are the computation resources effectively utilized?


Low resolution


High resolution


Low resolution


High resolution

Global LOD Quality Metric

- Measure the amount of information contained in the selected LOD
  - Compare LODs
  - Decide whether the computation resources are distributed evenly to render-worthy blocks
  - LOD adjustment
- Approach: Information theory

LOD Entropy

$$H(X) = -\sum_{i=1}^{M} p_i \log p_i$$

- A LOD contains a sequence of blocks B_i at particular resolutions
- P_i, the 'probability' of a data block B_i at a particular resolution, is defined as:

$$P_i = \frac{C_i \times D_i}{S} \qquad S = \sum_{i=1}^{M} C_i \times D_i$$

- C_i and D_i are the block's contribution and distortion (if it is a low resolution block)

Contribution and Distortion

- Contribution: the block's color (μ), projection size (a), thickness (t), visibility (v)

$$C_i = \mu.t.a.v$$

- Distortion: the difference between the block's data values and those of a higher resolution block

$$d_{ij} = \sigma_{ij} \cdot \frac{\mu_i^2 + \mu_j^2 + C_1}{2\mu_i\mu_j + C_1} \cdot \frac{\sigma_i^2 + \sigma_j^2 + C_2}{2\sigma_i\sigma_j + C_2}$$

covariance luminance contrast

LOD Entropy

$$H(X) = -\sum_{i=1}^{M} p_i \log p_i$$

- Maximize the entropy function when *Pi* are all equal
- The entropy function prefers that the block's contribution matches its resolution

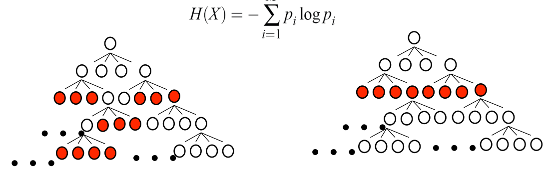
$$P_i = \frac{C_i \times D_i}{S}$$

Ci ↗ ⇨ Di ↘ : use high resolution

Ci ↘ ⇨ Di ↗ : use low resolution

LOD Comparisons using Entropy

$$H(X) = -\sum_{i=1}^{M} p_i \log p_i$$



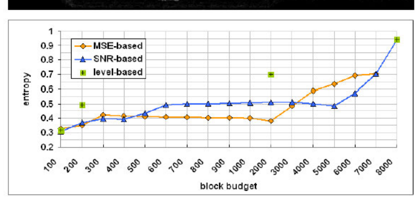
H1 H2

A higher entropy value indicates a balanced probability distribution, thus a better overall quality

Entropy vs. Quality

Entropy = 0.166 (34 blocks) Entropy = 0.316 (259 blocks)

Entropy vs. # of Blocks

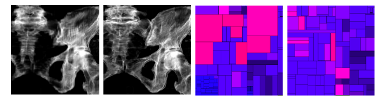


Visual Representation of LOD Quality

- An optimal selection of LOD is an NP complete problem
- Fine tuning of LOD selection is often necessary
- Can we visualize what are selected, and make adjustments if necessary?

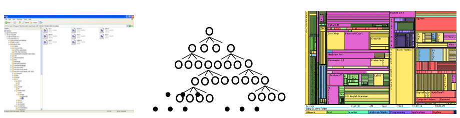
LOD Map

- A visual user interface to visualize the LOD selection
- Allow the user to see individual block's contribution vs. distortion, i.e., visualize the entropy terms



Treemap

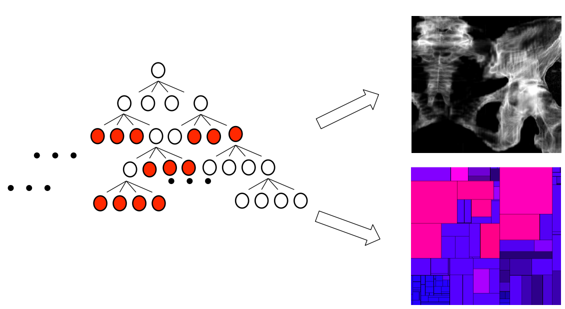
- A space-filling method to visualize hierarchical information [Shneiderman et al. 1992]
  - Recursive subdivision of a given display area
  - Information of each individual node
    - Color and size of its bounding rectangle



LOD Map

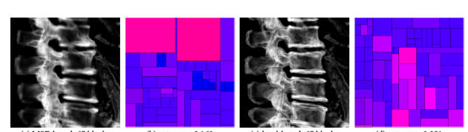
- Display the blocks belong to the selected LOD in a tree-map like manner
- Color (blue to red) is used to encode the block's distortion
- The contribution of the block (μ.t.a.v) is divided into two parts
  - The size of rectangle is to encode μ.t.a
  - The opacity of rectangle is to encode v

LOD Map



How Can LOD Map Help

Comparisons of different LOD selection schemes

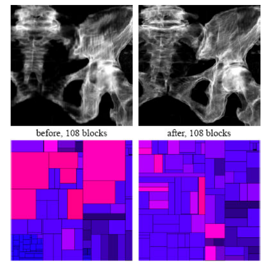


(a) MSE-based, 67 blocks (b) entropy = 0.163 (c) level-based, 67 blocks (d) entropy = 0.381

How Can LOD Map Help

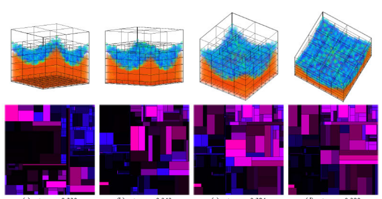
- Spot problematic regions in the current LOD
  - Large red rectangles – high contribution blocks rendered with low resolutions
    - Action: split the blocks and increase the resolutions
  - Small blue rectangles – low contribution blocks rendered with high resolutions
    - Action: join the blocks and reduce the resolutions
  - Dark rectangles – low visibility blocks
    - Action: join them and reduce the resolutions

LOD Adjustment



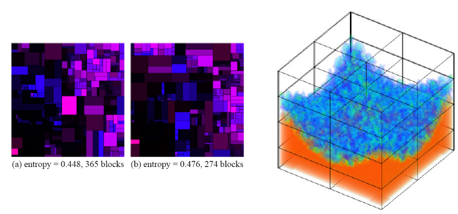
How Can LOD Map Help

- View selection on the fly - High entropy and brighter LOD map for better views


(a) entropy = 0.330 (b) entropy = 0.343 (c) entropy = 0.384 (d) entropy = 0.390

How Can LOD Map Help

- Budget Control - Render fewer blocks, i.e., lower Resolutions in certain regions, for the same entropy


(a) entropy = 0.448, 365 blocks (b) entropy = 0.476, 274 blocks

Conclusions

- Entropy can be used to quantify the information content in a visualization
- Applications: view selection & LOD selection
- The goal of visualization is to reduce the uncertainty perceived by the viewer
- More applications of information theory are expected to quantify the goodness of visualization

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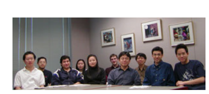

National Science Foundation WHERE DISCOVERIES BEGIN